International Journal of Electrical and Electronic Engineering & Telecommunications

ISSN 2319 – 2518 www.ijeetc.com
Vol. 5, No. 2, April 2016

Research Paper

ANALYTICAL METHOD FOR PATTERN GENERATION IN NINE-LEVEL CASCADED H-BRIDGE INVERTER USING SELECTIVE HARMONIC ELIMINATION

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This paper proposes an analytical procedure for computation of all pairs of valid switching angles used in pattern generation in nine-level H-bridge cascaded inverters. The proposed procedure eliminates harmonic components from inverter output voltage and, for each harmonic, returns the exact boundaries of all valid modulation index intervals. Due to its simple mathematical formulation, it can be easily implemented in real time using a digital signal processor or a field-programmable gate array. In this paper, after a detailed description of the method, simulation and experimental results demonstrate the high quality of achievable results.

Keywords: Cascaded multilevel inverters, Chebyshev polynomials, High-power converters, Modulation, Selective Harmonic Elimination (SHE), Total Harmonic Distortion (THD), Waring formulas

INTRODUCTION

Selective Harmonic Elimination (SHE) methods, originated by the harmonic elimination technique early pro-posed by Patel for high-power inverters, offer enhanced op-erations at low switching frequency while reducing size and cost of bulky passive filters [1], [2]. They have been suc-cessful adopted in different converter topologies, including cascaded H-bridge multilevel converters, whose N-level ac voltage outputs already improve the Total Harmonic Distortion (THD) [3]-[11].

SHE equations are nonlinear; moreover, simple, multiple, or even no solutions could be accomplished for each modulation index [11]. Moreover, it is necessary to know how to obtain all possible sets of solutions and where they exist [12], [13]. Once a set has been obtained, some selection criteria should be adopted. For instance, the lowest THD could be early identified and then selected [14]; another

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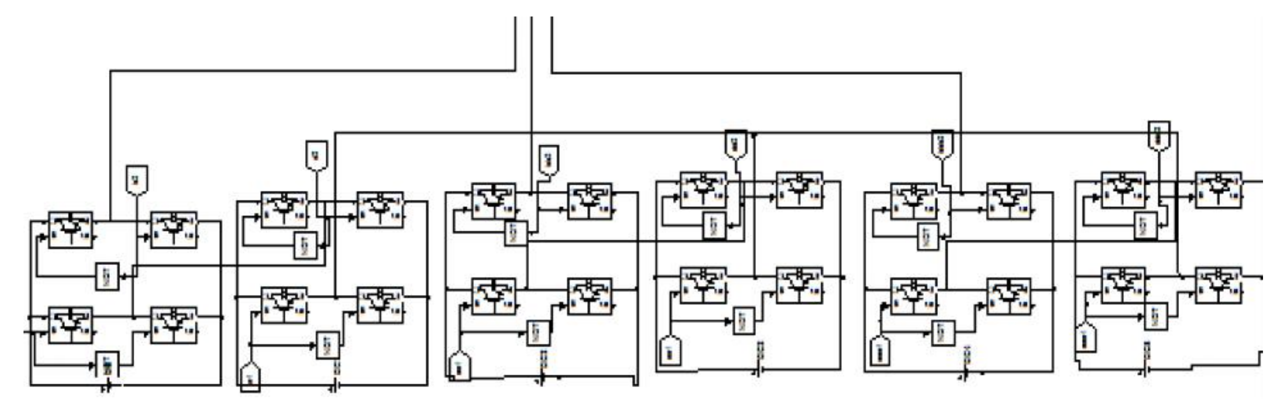
possible criterion could be minimization of mutations in solution sets [15], [16].

Many authors dealt with the SHE problem. The au-thors in [17]–[21] proposed iterative numerical techniques. The authors in [22] proposed a generalized formulation for half-cycle symmetry SHE–pulsewidth-modulation problems in multilevel inverters, solving the equations by using a Matlab function (*fsolve*). Authors in [20] proposed a harmonic suppres-sion technique for double-cell inverters called "mirror surplus harmonic method" and using an unconstrained optimization, and in [23], the Walsh function-based analytical technique is adopted, where the harmonic amplitude is expressed directly as a function of switching angles, the latter obtained solving linear algebraic equations rather than nonlinear transcenden-tal equations. In [24] and [25], theories of resultants and of symmetric polynomials were used to characterize, for each modulation index, the existence of the solution and then to solve polynomial equations obtained from transcendental equa-tions. Many authors use Genetic Algorithms (GAs) ([26]–[28]). In [29], the bee algorithm is applied and compared with a GA; in [30] and [31], a solution has been obtained by using particle swarm optimization (PSO). Paper [32] proposes a PSO-algorithm-based staircase modulation strategy for mod-ular multilevel converters. Bacterial foraging algorithm and ant colony method were adopted in [33] and [34], respec-tively. Another approach, based on homotopy and continua-tion theory, was proposed in [35]-[37] for determination of one set of solutions. Minimization techniques were proposed in [38] and [39].

The main drawbacks of existing SHE methods are their mathematical complexity and the heavy computational loads, resulting high cost of the hardware needed for real-time imple-mentation [10]. The last problem is commonly circumvented by preliminary off-line computation of the switching angles and the subsequent creation of lookup tables to be stored in the microcontroller's internal memory for real-time fetching. This approach, even if effective, in some cases could lead to some drawbacks, particularly the use of discrete modula-tion indices, resulting in nonoptimum commutations and the need of significant amounts of the microcontroller's memory. Some requirements cannot be fulfilled by current microcon-trollers/DSPs specifically designed for energy management and motion control [the so-called digital signal controllers (DSCs)], due to their limited amount of RAM and the rela-tively high speed of their internal FLASH memory [42], [43]. Moreover, implementation of feedback control requires distinct lookup tables for each modulation index and limits achievable

This paper considers five-level cascaded H-bridge inverters and proposes a procedure allowing fully analytical calculation (i.e., without any lookup table) of those switching angles ~1 and ~2, eliminating one harmonic *h*i (*i* = 3, 5, 7, …). The procedure uses *Chebyshev polynomials* and *Waring formulas* [40], [41] and, due to its limited complexity, can be easily im-plemented in real time using DSC, programmable logic device, or Field-Programmable Gate Array (FPGA) [42]. Moreover, it is almost general and can be extended to different topologies and, for the considered topology, to higher number of levels. Specific sets of equations have to be determined for each case. The proposed

Figure 1: Three-Phase Five-Level Cascaded Inverter (a) Schematic Diagram of a Three-Phase Five-Level Cascaded Inverter, (b) (A) Phase A and [(B) and (C)] Individual H-Bridge Output Waveforms Performance and/or Introduces Programming Complexity and/or Potential Problems in Closed-Loop Systems



method can be implemented both off-line and online, i.e., in real time. Its most important advantages are the possibility to obtain exact solution in real time and that it is possible to obtain all possible solutions.

The described features are very useful in implementation of closed-loop control and, due to limited algorithm complexity, do not require specific hardware [44].

In the following, Section II deals with problem formulation, Section III describes the determination of modulation index interval, and Section IV describes the procedure for the deter-mination of switching angles. Section V shows some simulation results and deals with some analysis, while Section VI reports experimental results and their comparison with simulations. Good agreement is noted among experimental and simulation results, confirming the accuracy of the proposed technique. Finally, Section VII draws some conclusions.

MODEL DESCRIPTION

A three-phase five-level cascaded inverter is considered. Its basic schematic diagram is shown in Figure 1a.

Each phase consists of two H-bridges fed by separate and balanced dc sources $vA1 = vA2 = Vd_c$.

Considering, for instance, phase *A* multilevel voltage: $vAN = vA1 + vA2$, its *Fourier series expansion* resultsthe following [45]:

$$v_{AN}(\omega t) = \frac{4V_{dc}}{\pi} \sum_{n} [\cos(n\alpha_1) + \cos(n\alpha_2)] \sin(n\omega t)/n \quad n = 1, 3, 5, 7, \ldots \tag{1}$$

where $\sim_1$ and $\sim_2$ are the switching angles necessary for mod-ulation. Figure 1b also shows modulation signals (b) and (c), which are necessary for obtaining the desired output waveform $vAN(\mu t)$.

The following condition has to be verified:

$$0 \leq \alpha_1 \leq \alpha_2 \leq \pi/2 \quad ...(2)$$

In order to eliminate a specific harmonic, the following condition must be imposed to find out the unknown switching angles $\sim_1$ and $\sim_2$:

$$\underline{F}(\underline{\alpha}) = \underline{0} \quad ...(3)$$

where $F = (F_1, F_2)$ and $\sim = (\sim_1, \sim_2)$ yield

$$\begin{cases} F_1(\alpha_1, \alpha_2) = \cos(\alpha_1) + \cos(\alpha_2) - 2m_1 \\ F_2(\alpha_1, \alpha_2) = \cos(k\alpha_1) + \cos(k\alpha_2) \end{cases} \quad ...(4)$$

and $k = 3, 5, 7, ...,$ is the harmonic order to be eliminated.

As shown in (4), the first equation in (3) is used to control the magnitude of the fundamental voltage, while the second one is used for harmonic order elimination. It is worth noticing from (4) that the number of equations corresponds to the number of existing H-bridges. The term m_1 represents the modulation index defined as $m1 = V_1/V_{1max}$, where V_1 is the fundamental output peak voltage and V_{1max} is the maximum obtainable fundamental peak voltage expressed as $V1_{max} = 8Vd_c/®$.

SEARCH OF MODULATION INDEX INTERVALS

Odd *Chebyshev polynomials* of first type T_k are introduced for the previously described problem and defined by recursive relationship [40]

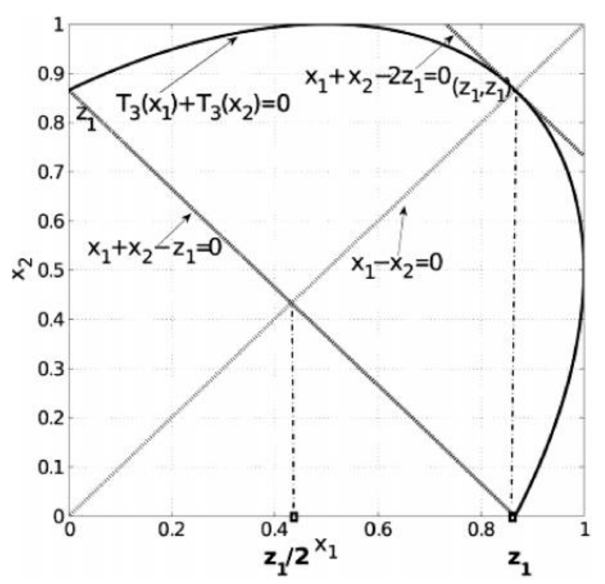
$$\begin{cases} T_0(x) = 1 \\ T_1(x) = x \\ T_{j+1}(x) = 2xT_j(x) - T_{j-1}(x) \end{cases} \quad ...(5)$$

or $T_j(x) = \cos(j \arccos(x))$, $j = 1, 2, ...$ The expressions $T_k(x)$ for $k = 3, 5, 7, 9, 11$, respectively, are described in the Appendix.

$$\begin{cases} T_1(x_1) + T_1(x_2) - 2m_1 = 0 \\ T_k(x_1) + T_k(x_2) = 0 \end{cases} \quad ...(6)$$

where $x_1 = \cos(\sim_1)$, $x_2 = \cos(\sim_2)$, and $k = 3, 5, 7, ...$ *Proposition 1:* Let $D = [0, 1] \times [0, 1]$, $zi = \cos(((2i - 1)/2k)®)$, $i = 1, ..., (k - 1)/2$, i.e., all positive zeros of the Chebyshev polynomial $T_k(x)$ with $k = 2l + 1$, $l > 1$.

Figure 2: Graphical Analysis for the Third-Harmonic Elimination



$$\begin{cases} x_1 + x_2 - 2m_1 = 0 \\ x_1 - x_2 = 0 \\ T_k(x_1) + T_k(x_2) = 0 \end{cases} \quad ...(7)$$

To better clarify the previous proposition, Figures 2 and 3d show the graphical representation of (7). The straight line $x_1 + x_2 - 2m_1 = 0$ is drawn at $m_1 = zi/2$ and $m1 = zi$, $i = 1, ..., (k - 1)/2$, for $k = 3, 5, 7, 9, 11$, respectively.

Theorem 1: The system (6) has real solutions in D if and only if m_1E[(1/2) cos(((k – 2)/2k)®), cos(®/2k)] c [0, 1].

Figures 2 and 3 show the whole set of solutions for (6) in D within different subintervals of modulation index m_1.

Considering, for instance, Figure 3a, the straight line $x_1 + x_2 = 2m_1$ obtained for the following.

1) $z2/2 < m_1 < z1/2$ intersects the curve $T5(x_1) + T5(x_2) = 0$ in two distinct symmetrical points (x_1, x_2) and (x_2, x_1), representing a

Figure 3: Graphical Analysis for the Harmonic Elimination, (a) Fifth Harmonic, (b) Seventh Harmonic, (c) Ninth Harmonic, (d) Eleventh Harmonic

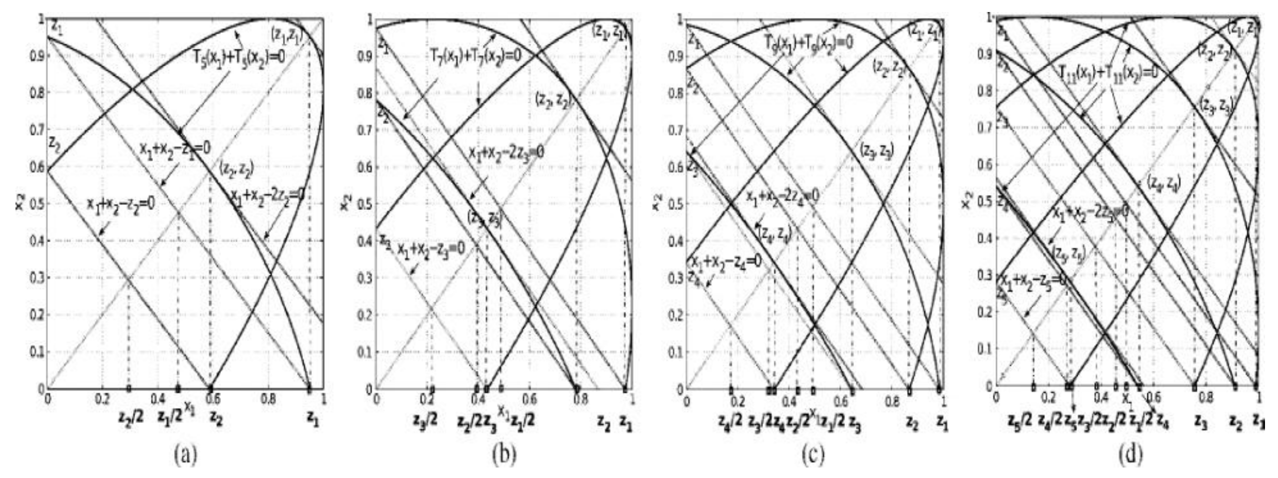
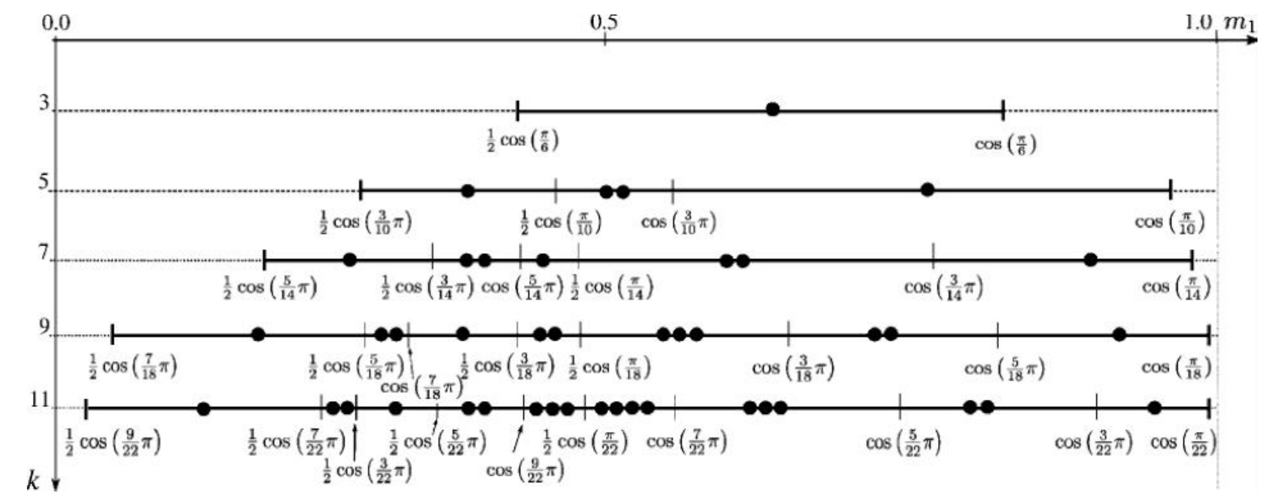


Figure 4: Valid Modulation Index Intervals to Vary Harmonic Order k to Eliminate



unique solution (for the symmetry) for the electrical problem;

2) $z1/2 < m_1 < z2$ intersects the curve $T5(x_1) + T5(x_2) = 0$ in four distinct symmetrical points, representing two solutions for the electrical problem;

3) $z2 < m_1 < z1$ intersects the curve $T5(x_1) + T5(x_2) = 0$ in two distinct symmetrical points, representing a unique solution for the electrical problem.

Similar considerations arise with any other harmonic.

Figure 4 shows valid modulation index intervals given by Theorem 1 at different k values (rows) and subintervals where different m_1's produce a different number of valid solutions $N_k(m_1)$. In the same figure, the ends of each subinterval are highlighted, and each dot within an interval represents a valid solution. The graphs in Figure 5 represent the functions $N_k(m_1)$, k = 3, 5, 7, 9, 11. Notice that the maximum number of solutions in a subinterval for each m_1 is $(k - 1)/2$ for $k = 3, 5$, and it is $(k - 3)/2$ for $k = 7, 9, 11$; hence, it is possible to know the number of solutions achieving the kth harmonic elimination ($k = 3, 5, ...,$).

DETERMINATION OF SWITCHING ANGLES

In order to explain the method in a simple way and without any loss of generality, in this paper, eliminations of third, fifth, and seventh harmonics are considered.

From (6), by applying the *Chebyshev polynomials* (see the Appendix), the following systems are obtained.

1) $k = 3$

$$\begin{cases} x_1 + x_2 = 2m_1 \\ 4\left(x_1^3 + x_2^3\right) - 3(x_1 + x_2) = 0. \end{cases} \tag{8}$$

2) $k = 5$

$$\begin{cases} x_1 + x_2 = 2m_1 \\ 16\left(x_1^5 + x_2^5\right) - 20\left(x_1^3 + x_2^3\right) + 5(x_1 + x_2) = 0. \end{cases} \tag{9}$$

3) $k = 7$

$$\begin{cases} x_1 + x_2 = 2m_1 \\ 64\left(x_1^7 + x_2^7\right) - 112\left(x_1^5 + x_2^5\right) + 56\left(x_1^3 + x_2^3\right) + \\ -7(x_1 + x_2) = 0. \end{cases} \tag{10}$$

Assuming that $s = x1 + x2$ and $p = x1x2$ and applying *Waring's formula* (see the Appendix) to previous systems, for each harmonic, the following equations in p with degree $l = (k$ " $1)/2$ can be obtained, respectively, with $s = 2m1$.

1) $k = 3$

$$12p - (4s^2 - 3) = 0. \tag{11}$$

2) $k = 5$

$$80p^2 + 20(3 - 4s^2)p + (16s^4 - 20s^2 + 5) = 0. \tag{12}$$

3) $k = 7$

$$448p^3 - 112(8s^2 - 5)p^2 + 56(8s^4 - 10s^2 + 3)p + \\ -(64s^6 - 112s^4 + 56s^2 - 7) = 0. \tag{13}$$

In the following, the acceptability of each solution for previ-ous equations is investigated.

In order to guarantee real solutions (x_1, x_2) E $[0, 1] \times [0, 1]$, for each valid modulation index, the following condition must be verified:

$$0 \le p \le m_1^2 \qquad \text{...(14)}$$

$$\begin{cases} x_1 + x_2 = 2m_1 \\ x_1 x_2 = p \end{cases}$$

The above function $P2(¶) = ¶^2 - s¶ + p$; hence, the modulator's angles $\sim_1$ and $\sim_2$ are computed by using inverse cosine function. No-tice that, regarding (14) with m_1 given by Theorem 1, condition $p \le m^2_1$ is obtained, imposing discriminant $s^2 - 4p \ge 0$ to solutions of $¶^2 - s¶ + p = 0$. It is worth noticing in Figures 2 and 3 that a relationship between m_1 and $(\sim_1, \sim_2)$ holds at the borders of each modulation index subinterval: when $m_1 = zi$, it fol-lows that $(\sim_1, \sim_2)$ = [arccos(zi), arccos(zi)], and when $m_1 = zi/2$, it follows that $(\sim_1, \sim_2)$ = [arccos(zi), ®/2]. Table 1 sum-marizes this relationship for the case shown in Figure 3a.

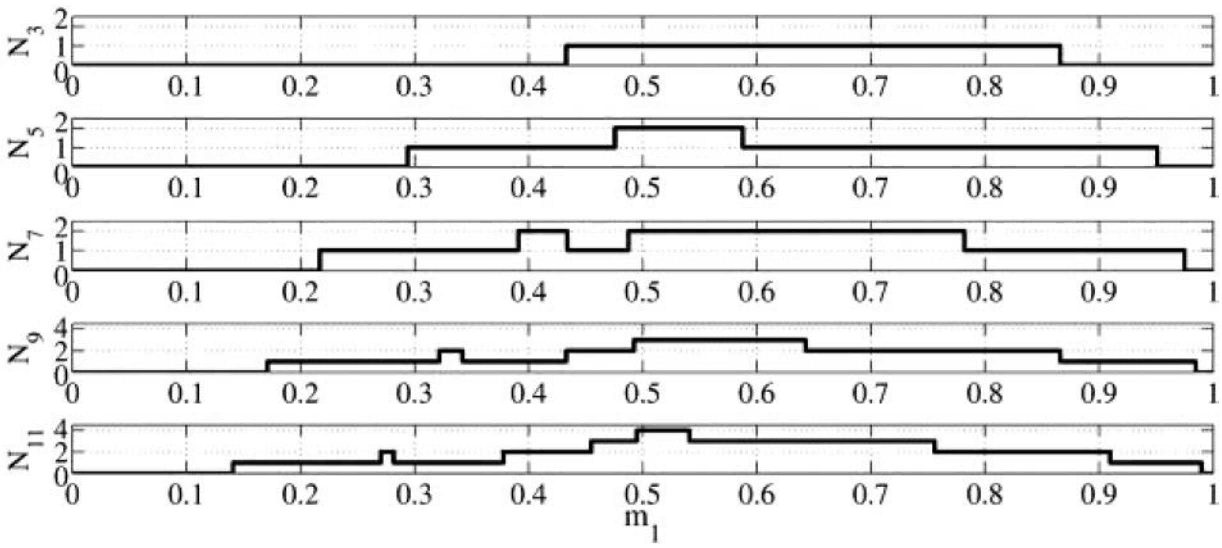
SIMULATION RESULTS

The proposed modulation technique was preliminarily ver-ified by means of

Table 1: Relationship Between m_1 and $(\sim_1, \sim_2)$ for the Fifth-Harmonic Elimination

m_1	α_1	α_2
$z_2/2$	$\arccos(z_2)$	$\pi/2$
$z_1/2$	$\arccos(z_1)$	$\pi/2$
z_2	$\arccos(z_2)$	$\arccos(z_2)$
z_1	$\arccos(z_1)$	$\arccos(z_1)$

Figure 5: Valid Modulation Index Intervals with the Solution (Switching Angle) Number



simulations and then implemented using a low-power simulation model. A complete converter with load was modeled using Matlab/Simulink [46]. SHE is typical of high power; therefore, simulations were carried out choosing a setup operating at medium voltage (Vd_c = 3 kV) and with rated power equal to 300 kVA. Load was simulated by an $R - L$ network consisting of a set of resistances R = 108 Ω in series with inductances L = 15 mH. Output phase voltage and corresponding Fast Fourier Transformation (FFT) spectrum are shown in Figure 6 for the third-, fifth-, and seventh-harmonic eliminations and modulation index equal to m_1 = 0.8, m_1 = 0.9, and m_1 = 0.8, respectively. It can be noticed that, in all cases, the desired harmonic is eliminated.

Figure 6: Three Phase Nine Level Output Voltage

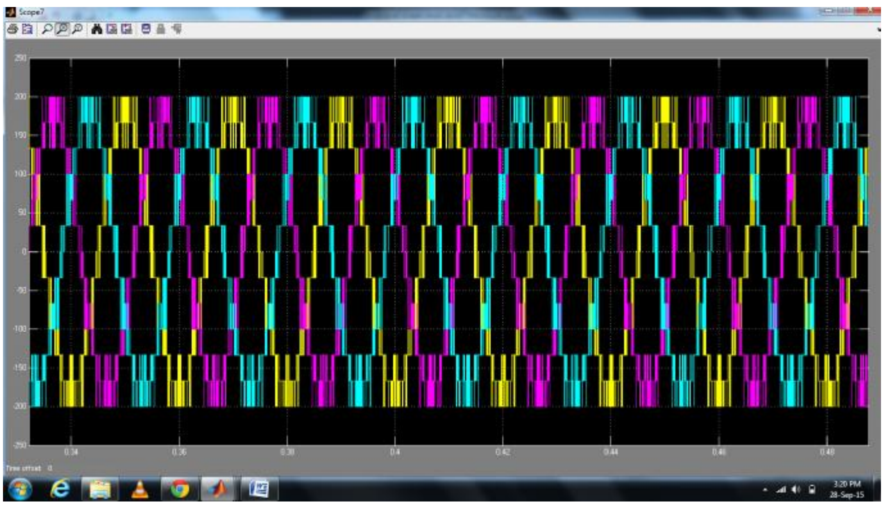


Figure 7: 3rd Harmonic

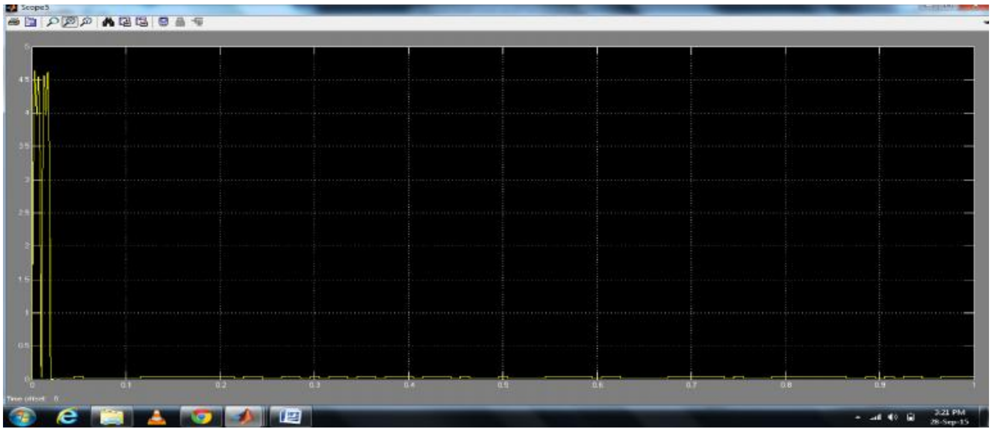


Figure 8: 5th Harmonic

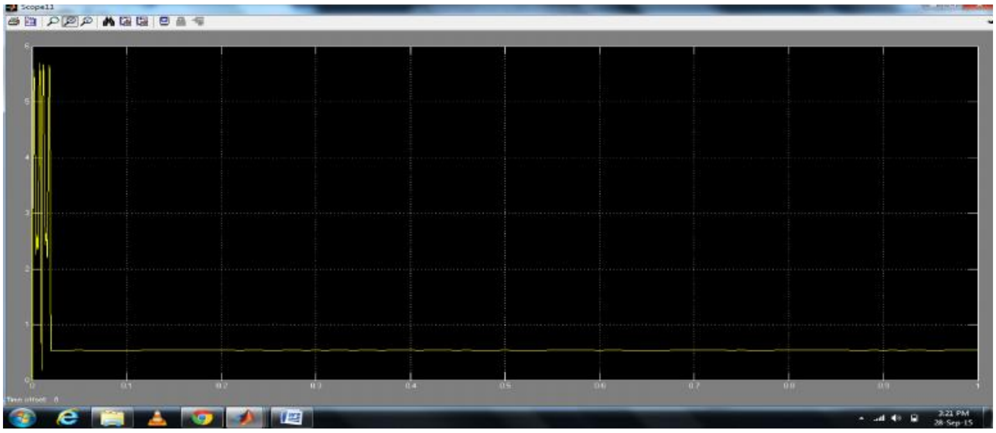


Figure 9: 7th Harmonic

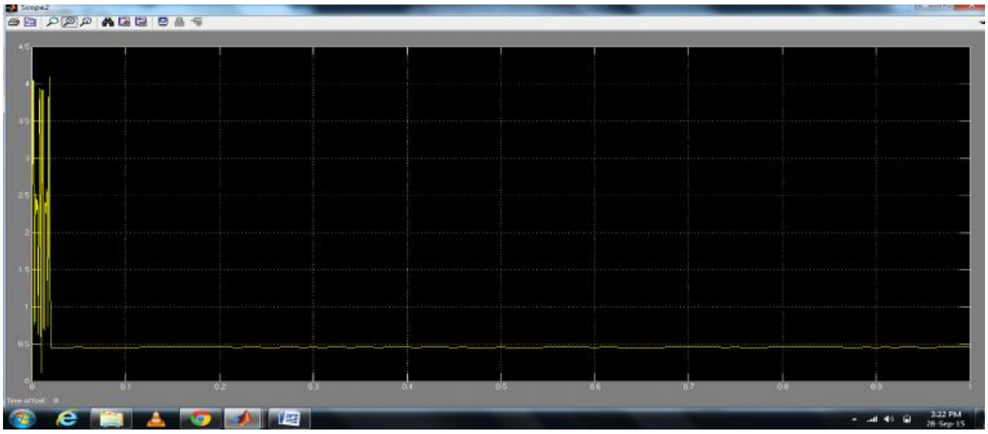


Figure 10: 9th Harmonic

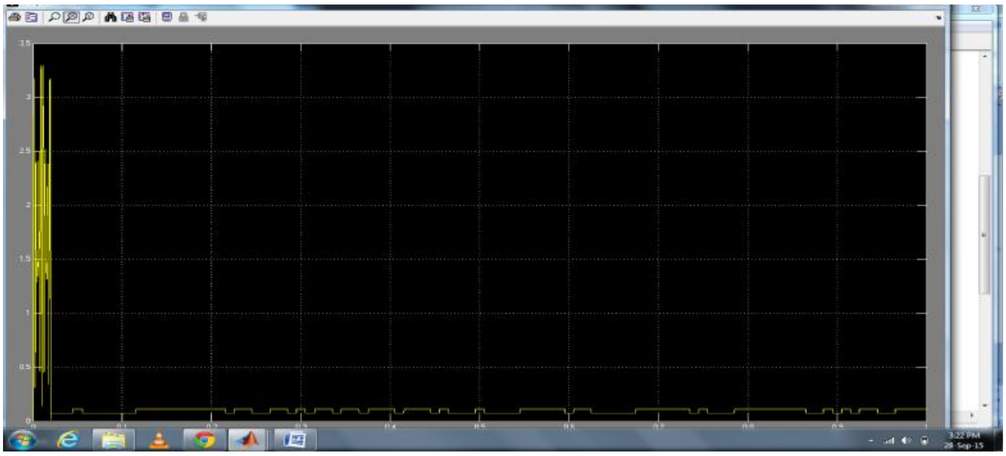
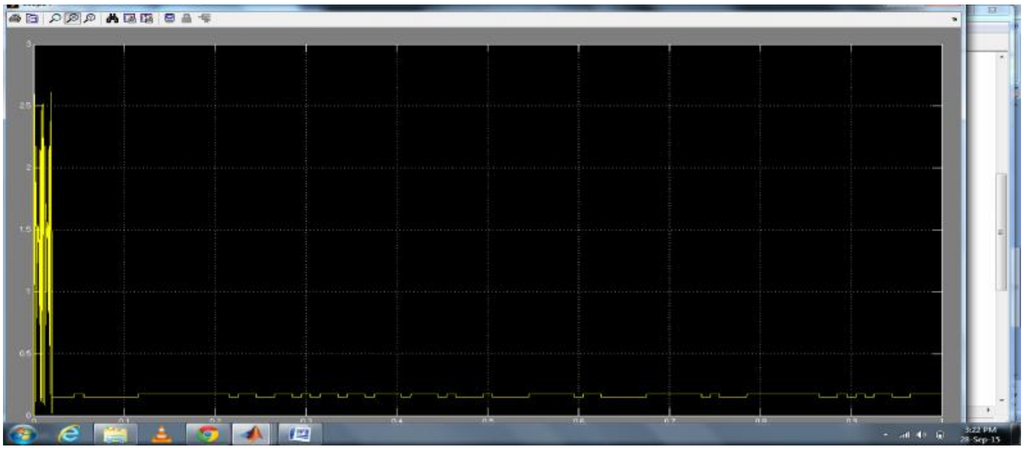


Figure 11: 11th Harmonic



CONCLUSION

In this paper, the problem of harmonic elimination in mul-tilevel converters has been addressed considering a five-level cascaded H-bridge inverter and proposing a new analytical algorithm for the computation of the switching angles $\sim_1$ and $\sim_2$ capable of elimination of harmonic signals. Since, in order to obtain these angles, it is necessary to find all valid intervals of the modulation index for which they exist, a new analytical procedure has been proposed, which returns the desired intervals split into subintervals and dependent on the number of pairs of switching angles capable of elimination of the selected harmonic. As an example, procedures for obtaining correct switching angles for third-, fifth-, and seventh-harmonic eliminations have been shown. The obtained simulation and experimental results confirm the accuracy of the proposed procedure. Work is in progress in order to extend the method to multiple harmonic elimination and to take into account multilevel voltage imbalances.

ACKNOWLEDGMENT

The authors would like to thank F Mancini, University of L'Aquila, for his assistance during laboratory activity, and the anonymous Associate Editor and Reviewers for their insightful comments.

REFERENCES

1. Patel H S and Hoft R G (1973), "Generalized Harmonic Elimination and Voltage Control in Thyristor Converters: Part I—Harmonic Elimination", *IEEE Trans. Ind. Appl.*, Vol. IA-9, No. 3, pp. 310-317.
2. Patel H S and Hoft R G (1974), "Generalized Harmonic Elimination and Voltage Control in Thyristor Converters: Part II—Voltage Control Technique", *IEEE Trans. Ind. Appl.*, Vol. IA-10, No. 5, pp. 666-673.
3. Napoles J, Watson A, Padilla J, Leon J, Franquelo L, Patrick W and Aguirre M (2013), "Selective Harmonic Mitigation Technique for Cascaded H-bridge Converters with Non-equal DC Link Voltages", *IEEE Trans. Ind. Electron.*, Vol. 60, No. 5, pp. 1963-1971.
4. Napoles J, Leon J I, Portillo R, Franquelo L G and Aguirre M A (2010), "Selective Harmonic Mitigation Technique for High-Power Converters", *IEEE Trans. Ind. Electron.*, Vol. 57, No. 7, pp. 2315-2323.
5. Franquelo L G, Napoles J, Portillo R, Leon J I and Aguirre M A (2007), "A Flexible Selective Harmonic Mitigation Technique to Meet Grid Codes in Three-Level PWM Converters", *IEEE Trans. Ind. Electron.*, Vol. 54, No. 6, pp. 3022-3029.
6. Gopakumar K, Azeez N A, Mathew J, Dey A and Kazmierkowski M (2014), "A Medium Voltage Inverter Fed IM Drive Using Multilevel 12-Sided Polygonal Vectors, with Nearly Constant Switching Frequency Current Hysteresis Controller", *IEEE Trans. Ind. Electron.*, Vol. 61, No. 4, pp. 1700-1709.
7. Sanzhong B and Lukic S M (2013), "New Method to Achieve AC Harmonic Elimination and Energy Storage Integration for 12-Pulse Diode Rectifiers", *IEEE Trans. Ind. Electron.*, Vol. 60, No. 7, pp. 2547-2554.
8. Filho F, Maia H Z, Mateus T H A, Ozpineci B, Tolbert L M and Pinto J O P (2013), "Adaptive Selective Harmonic Minimization Based on ANNs for Cascade Multilevel Inverters with Varying DC Sources", *IEEE Trans. Ind. Electron.*, Vol. 60, No. 5, pp. 1955-1962.
9. Pulikanti S R, Konstantinou G and Agelidis V G (2013), "Hybrid Seven-Level Cascaded Active Neutral-Point-Clamped-Based Multilevel Converter Under SHE-PWM", *IEEE Trans. Ind. Electron.*, Vol. 60, No. 11, pp. 4794-4804.
10. Guzman J I, Melin P E, Espinoza J R, Moran L A, Baier C R, Munoz J A and Guinez G A (2013), "Digital Implementation of Selective Harmonic Elimination Techniques in Modular Current Source Rectifiers", *IEEE Trans. Ind. Informat.*, Vol. 9, No. 2, pp. 1167-1177.
11. Lai J-S and Peng F Z (1996), "Multilevel Converters—A New Breed of Power Converters", *IEEE Trans. Ind. Appl.*, Vol. 32, No. 3, pp. 509-517.

12. Buccella C, Cecati C and Cimoroni M G (2012), "Investigation about Numerical Methods for Selective Harmonics Elimination in Cascaded Multilevel Inverters", in *Proc. Int. Conf. ESARS Propulsion*, October, pp. 1-6, Bologna, Italy.

13. Buccella C, Cecati C and Cimoroni M G (2013), "Harmonics Elimination in 5-Level Converters Operating at Very Low Switching Frequency", in *Proc. IEEE ICIT*, February 24-27, pp. 1946-1951, Cape Town, South Africa.

14. Peng F Z, Lai J-S, McKeever J W and Van Coevering J (1996), "A Multilevel Voltage-Source Inverter with Separate DC Sources for Static VAr Generation", *IEEE Trans. Ind. Appl.*, Vol. 32, No. 5, pp. 1130-1138.

15. Vassallo J, Clare J C and Wheeler P W (2003), "A Power-Equalized Harmonic-Elimination Scheme for Utility-Connected Cascaded H-bridge Multilevel Converters", in *Proc. IEEE 29th IECON/IECON*, Vol. 2, November 2-6, pp. 1185-1190.

16. Liu H L, Cho G H and Park S S (1995), "Optimal PWM Design for High Power Three-Level Inverter Through Comparative Studies", *IEEE. Trans. Power Electron.*, Vol. 10, No. 1, pp. 38-47.

17. Kumar J, Das B and Agarwal P (2008), "Selective Harmonic Elimination Technique for a Multilevel Inverter", in *Proc. 15th NPSC*, December, pp. 608-613, Bombay, India.

18. Tolbert L M, Peng F Z and Habetler T G (1999), "Multilevel Converters for Large Electric Drives", *IEEE Trans. Ind. Appl.*, Vol. 35, No. 1, pp. 36-44.

19. Sun J and Grotstollen I (1994), "Pulsewidth Modulation Based on Real-Time Solution of Algebraic Harmonic Elimination Equations", in *Proc. 20th IEEE IECON/IECON*, Vol. 1, September 5-9, pp. 79-84.

20. Li L, Czarkowski D, Liu Y and Pillay P (2000), "Multilevel Selective Harmonic Elimination PWM Technique in Series-Connected Voltage Inverters", *IEEE Trans. Ind. Appl.*, Vol. 36, No. 1, pp. 160-170.

21. Tang T, Han J and Tan X (2006), "Selective Harmonic Elimination for a Cascade Multilevel Inverter", in *Proc. IEEE ISIE*, Vol. 2, July, pp. 977-981.

22. Fei W, Du X and Wu B (2010), "A Generalized Half-Wave Symmetry SHE-PWM Formulation for Multilevel Voltage Inverters", *IEEE Trans. Ind. Electron.*, Vol. 57, No. 9, pp. 3030-3038.

23. Liang T J, O'Connell R M and Hoft R G (1997), "Inverter Harmonic Reduction Using Walsh Function Harmonic Elimination Method", *IEEE Trans. Power Electron.*, Vol. 12, No. 6, pp. 971-982.

24. Chiasson J N, Tolbert L M, McKenzie K J and Du Z (2003), "Control of a Multilevel Converter Using Resultant Theory", *IEEE Trans. Control Syst. Technol.*, Vol. 11, No. 3, pp. 345-353.

25. Chiasson J N, Tolbert L M, McKenzie K J and Du Z (2004), "A Unified Approach to Solving the Harmonic Elimination Equations in Multilevel Converters", *IEEE Trans. Power Electron.*, Vol. 19, No. 2, pp. 478-490.

26. Ozpineci B, Tolbert L M and Chiasson J N (2005), "Harmonic Optimization of Multilevel Converters Using Genetic Algorithms", *IEEE Power Electron. Lett.*, Vol. 3, No. 3, pp. 92-95.

27. Salehi R, Farokhnia N, Abedi M and Fathi S H (2011), "Elimination of Low Order Harmonics in Multilevel Inverter Using Genetic Algorithm", *J. Power Electron.*, Vol. 11, No. 2, pp. 132-139.

28. Dahidah M S A and Agelidis V G (2008), "Selective Harmonic Elimination PWM Control for Cascaded Multilevel Voltage Source Converters: A Generalized Formula", *IEEE Trans. Power Electron.*, Vol. 23, No. 4, pp. 1620-1630.

29. Kavousi A, Vahidi B, Salehi R, Bakhshizadeh M, Farokhnia N and Fathi S S (2012), "Application of the Bee Algorithm for Selective Harmonic Elimination Strategy in Multilevel Inverters", *IEEE Trans. Power Electron.*, Vol. 27, No. 4, pp. 1689-1696.

30. Hagh M T, Taghizadeh H and Razi K (2009), "Harmonic Minimization in Multilevel Inverters Using Modified Species-Based Particle Swarm Optimization", *IEEE Trans. Power Electron.*, Vol. 24, No. 10, pp. 2259-2267.

31. Kaviani A, Fathi S H, Farokhnia N and Ardakani A (2009), "PSO, an Effective Tool for Harmonics Elimination and Optimization in Multilevel Inverters", in *Proc. 4th IEEE Conf. Ind. Electron. Appl.*, May 25-27, pp. 2902-2907.

32. Shen K, Zhao D, Mei J, Tolbert L M, Wang J, Ban M, Ji Y and Cai X (2014), "Elimination of Harmonics in a Modular Multilevel Converter Using Particle Swarm Optimization-Based Staircase Modulation Strategy", *IEEE Trans. Ind. Electron.*, Vol. 61, No. 10, pp. 5311-5322.

33. Salehi R, Vahidi B, Farokhnia N and Abedi M (2010), "Harmonic Elimination and Optimization of Stepped Voltage of Multilevel Inverter by Bacterial Foraging Algorithm", *J. Elect. Eng. Technol.*, Vol. 5, No. 4, pp. 545-551.

34. Sundareswaran K, Jayant K and Shanavas T N (2007), "Inverter Harmonic Elimination Through a Colony of Continuously Exploring Ants", *IEEE Trans. Ind. Electron.*, Vol. 54, No. 5, pp. 2558-2565.

35. Kato T (1999), "Sequential Homotopy-Based Computation of Multiple Solutions for Selected Harmonic Elimination in PWM Inverters", *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, Vol. 46, No. 5, pp. 586-593.

36. Sun J, Beineke S and Grotstollen H (1996), "Optimal PWM Based on Real-Time Solution of Harmonic Elimination Equations", *IEEE Trans. Power Electron.*, Vol. 11, No. 4, pp. 612-621.

37. Xie Y X, Zhou L and Peng H (2000), "Homotopy Algorithm Research of the Inverter Harmonic Elimination PWM Model", *Proc. Chin. Soc. Elect. Eng.*, Vol. 20, No. 10, pp. 23-26.

38. Agelidis V G, Balouktsis A and Balouktsis I (2004), "On Applying a Minimization Technique to the Harmonic Elimination PWM Control: The Bipolar Waveform", *IEEE Power Electron. Lett.*, Vol. 2, No. 2, pp. 41-44.

39. Agelidis V G, Balouktsis A and Cosar C (2006), “Multiple Sets of Solutions for Harmonic Elimination PWM Bipolar Waveforms: Analysis and Experimental Verification”, *IEEE Trans. Power Electron.*, Vol. 21, No. 2, pp. 415-421.

40. Abramowitz M and Stegun I A (1964), *Handbook of Mathematical Functions*, National Bureau of Standards, Ser. Applied Mathematics Series 55, Washington, DC, USA.

41. Mac Donald I G (1995), *Symmetric Functions and Hall Polynomials*, Clarendon Press, Oxford, UK.

42. Buccella C, Cecati C and Latafat H (2012), “Digital Control of Power Converters—A Survey”, *IEEE Trans. Ind. Informat.*, Vol. 8, No. 3, pp. 437-447.

43. Cecati C (1999), “Microprocessors for Power Electronics and Electrical Drives Applications”, *IEEE Ind. Electron. Soc. Newslett.*, Vol. 46, No. 3, pp. 5-9.

44. Holtz J and Oikonomou N (2007), “Synchronous Optimal Pulsewidth Modulation and Stator Flux Trajectory Control for Medium-Voltage Drives”, *IEEE Trans Ind. Appl.*, Vol. 43, No. 2, pp. 600-608.

45. Escalante M F, Vannier J C and Arzande A (2002), “Flying Capacitor Multilevel Inverters and DTC Motor Drive Applications”, *IEEE Trans. Ind. Electron.*, Vol. 49, No. 4, pp. 809-815.

46. Matlab 7.13 (R2011b), *Software Package*, available at http://www.Mathworks.com

47. http://www.mcselec.com/?option=com_content &task=view&id=14&Itemid=41

48. www.xilinx.com/support/download/

APPENDIX

The $T_k(x)$ expressions, k = 3, ..., 11, are as follows.

1) $T_3(x) = 4x^3 - 3x$.

2) $T_5(x) = 16x^5 - 20x^3 + 5x$.

3) $T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$.

4) $T_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$.

5) $T_{11}(x) = 1024x^{11} - 2816x^9 + 2816x^7 - 1232x^5 + 220x^3 - 11x$.

Proof of Proposition 1

The straight line $x_1 + x_2 = 2m_1$ with $m_1 = zi/2$, i = 1, ..., (k – 1)/2, intersects boundaries $x_1 = 0$ and $x_2 = 0$ of Din (zi, 0) and (0, zi) for i = 1, ..., (k – 1)/2. These points (zi, 0) and (0, zi) are also solutions of the equation $T_k(x_1) + T_k(x_2) = 0$. If (x_1, x_2) D, it is easy to verify that the intersection of curve $T_k(x_1) + T_k(x_2) = 0$ with straight line $x_1 \downarrow x_2 = 0$ is the point (zi, zi) satisfying also equation $x_1 + x_2 \downarrow 2zi = 0$.

Proof of Theorem 1

Considering thatmin {zi} = z(k-1)/2 = cos(((k - ↓

$1\delta_i$¢÷(k –1)/2

2)/2k)π) and max1δiδ(k – 1)/2{zi} = z1 = cos(π/2k), by *Proposition 1* and graphical analysis results, the thesis follows. The *Waring formulas* for the considered harmonics with $s = x_1 + x_2$ and $p = x_1x_2$ are as follows:

$x^3_1 + x_2^3 = s^3 \downarrow 3ps$

$x^5_1 + x^52 = s^5 \downarrow 5ps^3 + 5p^2s$

$x^7_1 + x^7_2 = s^7 \downarrow 7ps^5 + 14p^2s^3 – 7p^3s$