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Research Paper

EQUIVALENT TRANSFER FUNCTION BASED DECOUPLER FOR A 4 x 4 DISTILLATION COLUMN MODEL

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Distillation is the most important separation method in the chemical and petrochemical industries. Distillation Column is highly non-linear multivariable process so it is very difficult to control such a complex process without eliminating the interactions, therefore in order to completely eliminate these interactions a Decoupler is introduced.

Keywords: Decoupler, Distillation column, Liquid-vapor, Single-Input Single-Output (SISO), Model-Reference Adaptive Control (MRAC)

INTRODUCTION

Distillation is a process which is used in petroleum industries for separation of products into an overhead distillate and a bottoms product. The distillation column is highly interactive multivariable process, so it is very difficult to control such a complex process without eliminating the interactions, therefore in order to completely eliminate these interactions a Decoupler is necessarily important. This paper is mainly divided into two parts: first, to present a theoretical calculation procedure for a distillation column which is highly non-linear for simulation and analysis and second, a controller design using Decoupler.

In this paper, a procedure for modeling and control of distillation column based on the energy balance (Liquid-Vapor) structure is introduced. In this control, the reflux rate 'L' and the boil-up rate 'V' are used as the inputs to control the outputs of the purity of the distillate overhead and the impurity of the bottom products.

In this paper, the modeling and simulation of distillation column is accomplished over three phases:

1. The basic nonlinear model of the plant,
2. The full-order linearized model, and
3. The reduced-order linear model.

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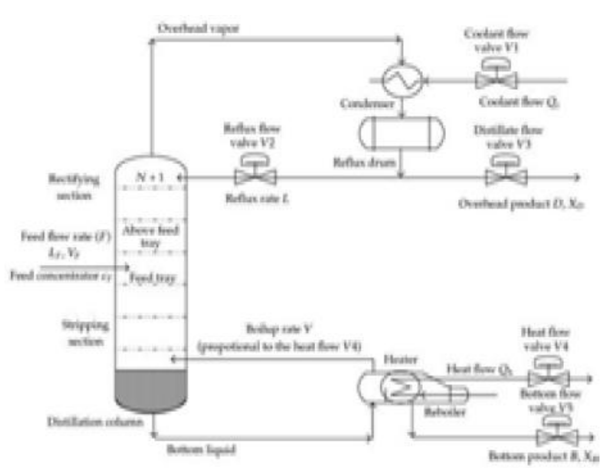
The reduced-order linear model is then used as the reference model for a Model-Reference Adaptive Control (MRAC). A reduced-order linear model is derived such that it best reflects the dynamics of the distillation process and used as the reference model for a Model-Reference Adaptive Control (MRAC).A decoupling control system design for high-dimensional multi-input, multi-output (MIMO) processes is discussed and solved. Based on the Relative Normalized Gain Array (RNGA), an Equivalent Transfer Function (ETF) for each element in the transfer function matrix was derived for the closed-loop control system and was used to approximate the inverse of the process transfer function matrix. The decoupler could be easily determined with each element in the First-Order-plus-Time-Delay (FOPTD) form and resulted in a stable, proper and causal decoupled matrix. A PI/PID controller could then be designed to meet the performance objectives. The main advantage of this method was its simplicity; it did not require extensive calculation effort.

DECOUPLER DESIGN METHODOLOGY

In the process control industry, more than 95% of loops are controlled by PI/PID controllers because of their relative effectiveness and their simple structure, which can be easily understood and implemented by practical engineers (Astrom and Hagglund, 1995). Consequently, PID control-algorithm development and application is still an actively researched area. However, the requirements for high product quality, material integration and energy integration have resulted in closely coupled variables for most modern industrial processes. This coupling has rendered many of the established Single-Input, Single-Output (SISO) PID tuning techniques insufficient for dealing with these multi-input, multi-output (MIMO) processes (Grosdidier and Morari, 1987). Adjusting the controller parameters of one loop affects the performance of the other loops, sometimes to the extent of destabilizing the entire system.

Although considerable effort has been dedicated to this problem and many design techniques have been proposed, multivariable control system design and implementation is still a difficult problem for control engineers because of the lack of practical approaches. Since controllers interact with each other in MIMO processes, the performance of one loop cannot be evaluated without information on the controllers in other loops. To solve this problem, a decoupling control scheme has been proposed that introduces a decoupler to eliminate the effect of interactions. The MIMO process can then be treated as multiple SISO loops, and less conservative single-loop control design methods can be directly applied. Therefore, when the control loops are closely coupled and tight control is required,

Figure 1: Distillation Column



decoupling control strategies are preferred in engineering practice.

A general decoupling control system contains the n-dimensional process matrix, the decoupler matrix and the controller transfer function matrix, respectively. $G_I(s)$ acts upon the process, $G(s)$, such that the transfer function matrix $G_R(s) = G(s)\ G_I(s)$ is a stable, proper and causal diagonal transfer matrix.

Decoupling Control for High-Dimensional MIMO Processes

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In this paper, the normalized decoupling control design methodology is extended from a 2 x 2 design to a high-dimensional MIMO design. The design is based on the concept of Relative Normalized Gain Array (RNGA) design. Using the gain and phase information revealed in RNGA design, the Equivalent Transfer Function (ETF) of each element in the transfer function matrix is derived for the closed-loop control system. Further, an equivalent transfer function matrix was approximated by the inverse of the process transfer function matrix used for calculating decoupling matrix. The decoupler could then be easily determined by multiplying the inverse of the ETF with a stable, proper and causal ideal-diagonal transfer function. Finally, a PI/PID controller is designed for the diagonal transfer function to meet the control system's performance target. Since almost all industry processes are open-loop stable and exhibit non-oscillatory behavior for step inputs, the higher order transfer function elements in the matrix, $G(s)$, can be simplified either by analytical or by empirical methods to a First-Order-Plus-Time-Delay (FOPTD) model for interaction analysis and control system design.

$$g_{ij}(s) = \frac{k_{ij}}{\tau_{ij}s+1}e^{-\theta_{ij}s}, i, j = 1, 2, \ldots n \qquad \ldots(1)$$

To describe the dynamic properties of a transfer function, the normalized gain, $K_{N,ij}$, for a particular transfer function, $g_{ij}(s)$ and the normalized gain matrix for the overall process, $G(s)$, are defined as:

$$K_{N,ij} \triangleq \frac{k_{ij}}{\sigma_{ij}} = \frac{k_{ij}}{\tau_{ij}+\theta_{ij}}, i, j = 1, 2, \ldots n \qquad \ldots(2)$$

and

$$K_N = \begin{bmatrix} k_{N,11} & k_{N,12} & \cdots & k_{N,1n} \\ k_{N,21} & k_{N,22} & \cdots & k_{N,2n} \\ \vdots & \vdots & \vdots & \vdots \\ k_{N,n1} & k_{N,n2} & \cdots & k_{N,nn} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{k_{11}}{\tau_{11}+\theta_{11}} & \frac{k_{12}}{\tau_{12}+\theta_{12}} & \cdots & \frac{k_{1n}}{\tau_{1n}+\theta_{1n}} \\ \frac{k_{21}}{\tau_{21}+\theta_{21}} & \frac{k_{22}}{\tau_{22}+\theta_{22}} & \cdots & \frac{k_{2n}}{\tau_{2n}+\theta_{2n}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{k_{n1}}{\tau_{n1}+\theta_{n1}} & \frac{k_{n2}}{\tau_{n2}+\theta_{n2}} & \cdots & \frac{k_{nn}}{\tau_{nn}+\theta_{nn}} \end{bmatrix} \qquad \ldots(3)$$

respectively. In the above expressions, is the average residence time of loops *i-j*.

Based on the normalized gain, the RNGA can be calculated as

$$\Phi = K_N \otimes K_N^{-T} \qquad \ldots(4)$$

where

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \qquad \ldots(5)$$

and the operator, $\otimes$, is the Hadamard product.

Furthermore, the relative average residence time, γ_{ij}, which is defined as the ratio of loop $y_i - u_j$ average residence time between when other loops are closed and when other loops are open can be obtained by

$$\gamma_{ij} = \frac{\hat{\sigma}_{ij}}{\sigma_{ij}} = \frac{\phi_{ij}}{\lambda_{ij}}, i, j = 1, 2, \ldots n \qquad \ldots(6)$$

where λ_{ij} is the element of the Relative Gain Array (RGA)

$$\lambda_{ij} = \frac{k_{ij}}{\hat{k}_{ij}}, i, j = 1, 2, \ldots n \qquad \ldots(7)$$

When the relative average residence times are calculated for all of the input/output combinations of a multivariable process, the results are in an array form, the Relative Average Residence Time Array (RARTA).

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix} \qquad \ldots(8)$$

Which is calculated by

$$\Gamma = \Phi \oslash \Lambda = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \oslash \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{bmatrix} \qquad \ldots(9)$$

where the operator, is the Hadamard division.

Since the relative average residence time is the ratio of the average residence times between when other loops are closed and when other loops are open, γ_{ij} represents the dynamic changes of the transfer function, $g_{ij}(s)$, when other loops are closed. Using the definition of RARTA, It is possible to write

$$\hat{\pm}_{ij} = ¥_{ij} \times \pm_{ij} = ¥_{ij} \times {}^{2}{}_{ij} + ¥_{ij} \times {}^{-}{}_{ij},$$

for i, j = 1, 2, …, n ...(10)

By using information from RGA, RNGA and RARTA, it is possible to uniquely determine the gain and the phase changes of a transfer function element when other loops closed. That is, a transfer function element of a MIMO process when other loops are closed can be approximated by a transfer function element having the same form as the open-loop transfer function element. However, the steady state gain, the time constant and the time delay are scaled by RGA and RARTA, respectively.

$$\hat{g}_{ij}(s) = \hat{k}_{ij} \times \frac{1}{\hat{{}^{2}}_{ij}\, s+1} e^{-\hat{{}^{-}}_{ij} s} = \frac{k_{ij}}{ª_{ij}} \times \frac{1}{¥_{ij}{}^{2}{}_{ij} s+1} e^{-¥_{ij}{}^{-}{}_{ij} s}$$

for i, j = 1, 2, …, n ...(11)

where $\hat{g}_{ij}(s)$ is the optimal ETF of loop *i-j* when other loops are closed (under the IE criterion). Therefore, $\hat{g}_{ij}(s)$ should resample the dynamic response of the corresponding true transfer function element when other loops are closed.

The relationship between $G^{-1}(s)$ and $\hat{G}^{T}(s)$

The dynamic Relative Gain Array (dRGA) is:

$$\Lambda(s) = G(s) \otimes \hat{G}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} \otimes \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{12}(s) & \cdots & 1/\hat{g}_{1n}(s) \\ 1/\hat{g}_{21}(s) & 1/\hat{g}_{22}(s) & \cdots & 1/\hat{g}_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ 1/\hat{g}_{n1}(s) & 1/\hat{g}_{n2}(s) & \cdots & 1/\hat{g}_{nn}(s) \end{bmatrix}$$

...(12)

where

$$\hat{G}(s) = \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{12}(s) & \cdots & 1/\hat{g}_{1n}(s) \\ 1/\hat{g}_{21}(s) & 1/\hat{g}_{22}(s) & \cdots & 1/\hat{g}_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ 1/\hat{g}_{n1}(s) & 1/\hat{g}_{n2}(s) & \cdots & 1/\hat{g}_{nn}(s) \end{bmatrix}$$

...(13)

However, for ideal control it is possible to write

$$u(s) = G^{-1}(s)\, y(s)$$ ...(14)

where the gain from $u_j(s)$ to $y_i(s)$ is $1/[G^{-1}(s)]_{ji}$ when all other outputs are perfectly controlled. Thus, the dRGA can be computed using the formula

$$\Lambda(s) = G(s) \otimes G^{T}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} \otimes \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix}^{-T}$$

...(15)

Comparing Equations (12) and (15) results in the following:

$$\begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix}^{-T} = \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{12}(s) & \cdots & 1/\hat{g}_{1n}(s) \\ 1/\hat{g}_{21}(s) & 1/\hat{g}_{22}(s) & \cdots & 1/\hat{g}_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ 1/\hat{g}_{n1}(s) & 1/\hat{g}_{n2}(s) & \cdots & 1/\hat{g}_{nn}(s) \end{bmatrix}$$

...(16)

Taking transpose of both sides of Equation (16) obtains

$$G^{-1}(s) = G^{\hat{T}}(s)$$ ...(17)

The definition of the matrix, $\hat{k}$ is

$$\hat{K} = \begin{bmatrix} \hat{k}_{11}^{-1} & \hat{k}_{12}^{-1} & \cdots & \hat{k}_{1n}^{-1} \\ \hat{k}_{21}^{-1} & \hat{k}_{22}^{-1} & \cdots & \hat{k}_{2n}^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{k}_{n1}^{-1} & \hat{k}_{n2}^{-1} & \cdots & \hat{k}_{nn}^{-1} \end{bmatrix} \quad ...(18)$$

Decoupling Control System Design

By substituting $\hat{G}^{T}(s)$, the design of an ideal-diagonal decoupler problem is transformed to determine the decoupler, $G_I(s)$

$$G_I(s) = \hat{G}^{T}(s) G_R(s) \quad ...(19)$$

A diagonal matrix, $G_R(s)$, is specified such that Equation (26) holds. Consequently, the design of the normalized decoupler started from the obtained $\hat{G}^{T}(s)$ determined the diagonal forward transfer function matrix, $G_R(s)$, such that the decoupler, $G_I(s)$, from Equation (19) satisfied realizable conditions.

From Equation (19) it can be seen that

$$G_I(s) = \hat{G}^{T}(s) G_R(s) \Rightarrow \begin{bmatrix} g_{I,11}(s) & g_{I,12}(s) & \cdots & g_{I,1n}(s) \\ g_{I,21}(s) & g_{I,22}(s) & \cdots & g_{I,2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{I,n1}(s) & g_{I,n2}(s) & \cdots & g_{I,nn}(s) \end{bmatrix}$$

$$= \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{21}(s) & \cdots & 1/\hat{g}_{n1}(s) \\ 1/\hat{g}_{12}(s) & 1/\hat{g}_{22}(s) & \cdots & 1/\hat{g}_{n2}(s) \\ \vdots & \vdots & \vdots & \vdots \\ 1/\hat{g}_{1n}(s) & 1/\hat{g}_{2n}(s) & \cdots & 1/\hat{g}_{nn}(s) \end{bmatrix} \times \begin{bmatrix} g_{R,11}(s) & 0 & \cdots & 0 \\ 0 & g_{R,22}(s) & \cdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \cdots & g_{R,nn}(s) \end{bmatrix} \quad ...(20)$$

Which results in

$$\begin{bmatrix} g_{I,11}(s) & g_{I,12}(s) & \cdots & g_{I,1n}(s) \\ g_{I,21}(s) & g_{I,22}(s) & \cdots & g_{I,2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{I,n1}(s) & g_{I,n2}(s) & \cdots & g_{I,nn}(s) \end{bmatrix} = \begin{bmatrix} \frac{g_{R,11}(s)}{\hat{g}_{11}(s)} & \frac{g_{R,22}(s)}{\hat{g}_{21}(s)} & \cdots & \frac{g_{R,nn}(s)}{\hat{g}_{n1}(s)} \\ \frac{g_{R,11}(s)}{\hat{g}_{12}(s)} & \frac{g_{R,22}(s)}{\hat{g}_{22}(s)} & \cdots & \frac{g_{R,nn}(s)}{\hat{g}_{n2}(s)} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{g_{R,11}(s)}{\hat{g}_{1n}(s)} & \frac{g_{R,22}(s)}{\hat{g}_{2n}(s)} & \cdots & \frac{g_{R,nn}(s)}{\hat{g}_{nn}(s)} \end{bmatrix} \quad ...(21)$$

To see how the problem's definition and the design procedure of a normalized decoupling control system is different from the existing methods, each element of the process transfer function matrix, the ETF, and the desired forward transfer function elements were of the form

$$g_{R,ii}(s) = \frac{e^{-\theta_{R,ii} s}}{\tau_{R,ii} s + 1} \quad i, j = 1, 2, \ldots, n \quad ...(22)$$

where $\tau_{R,ii}$ and $\theta_{R,ii}$ are the adjustable time constant and the dead time of $g_{R,ii}(s)$, respectively.

By substituting Equation (22) into (21), the element in the ideal decoupler matrix has the form

$$g_{I,ij}(s) = \frac{e^{-\left(\theta_{R,ii} - \hat{\theta}_{ji}\right)s}}{\hat{k}_{ji}} \times \frac{\hat{\tau}_{ji} s + 1}{\tau_{R,ii} s + 1} \quad i, j = 1, 2, \ldots, n$$

...(23)

CONTROL SYSTEM DESIGN

The dynamics of the Distillation Column process is modeled by using the closed-loop identification method as 2 x 2, 3 x 3, 4 x 4 MIMO process in the literature [6]. In this paper we considered the mathematical model of a 4 x 4 MIMO process for the decoupler design [2].

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} \frac{-0.098}{122s+1}e^{-17s} & \frac{-0.036}{149s+1}e^{-27s} & \frac{-0.014}{158s+1}e^{-32s} & \frac{-0.017}{155s+1}e^{-30s} \\ \frac{-0.043}{147s+1}e^{-25s} & \frac{-0.092}{130s+1}e^{-16s} & \frac{-0.011}{156s+1}e^{-33s} & \frac{-0.012}{157s+1}e^{-34s} \\ \frac{-0.012}{153s+1}e^{-31s} & \frac{-0.016}{151s+1}e^{-34s} & \frac{-0.102}{118s+1}e^{-16s} & \frac{-0.033}{146s+1}e^{-26s} \\ \frac{-0.013}{156s+1}e^{-32s} & \frac{-0.015}{159s+1}e^{-31s} & \frac{-0.029}{144s+1}e^{-25s} & \frac{-0.108}{128s+1}e^{-18s} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

...(24)

where *u*1, *u*2, *u*3 and *u*4 are the manipulated variables, and *T*1, *T*2, *T*3 and *T*4 are the controlled variables.

Using the formulas provided above, the normalized gain matrix (K), the RGA (Λ), the RNGA (Φ) and the RARTA (Γ) could be calculated as:

$$K = 1.0\text{e-}003 \times \begin{bmatrix} -0.7050 & -0.2045 & -0.0737 & -0.0919 \\ -0.2500 & -0.6301 & -0.0582 & -0.0628 \\ -0.0625 & -0.0865 & -0.7612 & -0.1919 \\ -0.0691 & -0.0789 & -0.1716 & -0.7397 \end{bmatrix} \quad ...(25)$$

$$\Lambda = \begin{bmatrix} 1.2207 & -0.2051 & -0.0053 & -0.0103 \\ -0.1947 & 1.2198 & -0.0136 & -0.0116 \\ -0.0106 & -0.0085 & 1.1095 & -0.0904 \\ -0.0154 & -0.0062 & -0.0907 & 1.1124 \end{bmatrix} \quad ...(26)$$

$$\Phi = \begin{bmatrix} 1.1389 & -0.1287 & -0.0036 & -0.0067 \\ -0.1238 & 1.1389 & -0.0078 & -0.0073 \\ -0.0057 & -0.0055 & 1.0710 & -0.0597 \\ -0.0094 & -0.0046 & -0.0596 & 1.0737 \end{bmatrix} \quad ...(27)$$

and

$$\Gamma = \begin{bmatrix} 0.9330 & 0.6274 & 0.6821 & 0.6461 \\ 0.6361 & 0.9336 & 0.5731 & 0.6269 \\ 0.5404 & 0.6532 & 0.9653 & 0.6604 \\ 0.6083 & 0.7460 & 0.6575 & 0.9652 \end{bmatrix} \quad ...(28)$$

The ETF parameters were

$$\hat{K} = \begin{bmatrix} -0.0803 & 0.1755 & 2.6653 & 1.6468 \\ 0.2209 & -0.0754 & 0.8097 & 1.0363 \\ 1.1315 & 1.8867 & -0.0919 & 0.3648 \\ 0.8420 & 2.4132 & 0.3197 & -0.0971 \end{bmatrix} \quad ...(29)$$

$$\hat{T} = \begin{bmatrix} 113.8299 & 93.4854 & 107.7763 & 100.1438 \\ 93.5129 & 121.3735 & 89.4106 & 98.4299 \\ 82.6888 & 98.6274 & 113.9022 & 96.4196 \\ 94.8909 & 118.6090 & 94.6824 & 123.5480 \end{bmatrix} \quad ...(30)$$

$$\hat{L} = \begin{bmatrix} 15.8615 & 16.9403 & 21.8181 & 19.3827 \\ 15.9036 & 14.9383 & 18.9138 & 21.3160 \\ 16.7539 & 22.2075 & 15.4444 & 17.1706 \\ 19.4648 & 23.1250 & 16.4379 & 17.3739 \end{bmatrix} \quad ...(31)$$

Which gives

$$\hat{G}^{T}(s) = \begin{bmatrix} \frac{113.8299s+1}{-0.0803}e^{15.8615s} & \frac{93.5129s+1}{0.2209}e^{15.9036s} & \frac{82.6888s+1}{1.1315}e^{16.7539s} & \frac{94.8909s+1}{0.8420}e^{19.4648s} \\ \frac{93.4854s+1}{0.1755}e^{16.9403s} & \frac{121.3735s+1}{-0.0754}e^{14.9383s} & \frac{98.6274s+1}{1.8867}e^{22.2075s} & \frac{118.6090s+1}{2.4132}e^{23.1250s} \\ \frac{107.7763s+1}{2.6653}e^{21.8181s} & \frac{89.4106s+1}{0.8097}e^{18.9138s} & \frac{113.9022s+1}{-0.0919}e^{15.4444s} & \frac{94.6824s+1}{0.3197}e^{16.4379s} \\ \frac{100.1438s+1}{1.6468}e^{19.3827s} & \frac{98.4299s+1}{1.0363}e^{21.3160s} & \frac{96.4196s+1}{0.3648}e^{17.1706s} & \frac{123.5480s+1}{-0.0971}e^{17.3739s} \end{bmatrix} \quad ...(32)$$

Using the normalized decoupling control system design rules proposed earlier, the decoupled forward transfer function is selected as

$$G_R(s) = \begin{bmatrix} \frac{e^{-21.8181s}}{113.8299s+1} & & & \\ & \frac{e^{-21.3160s}}{121.3735s+1} & & \\ & & \frac{e^{-22.2075s}}{113.9022s+1} & \\ & & & \frac{e^{-23.1250s}}{123.5480s+1} \end{bmatrix} \quad ...(33)$$

This gives a stable, causal and proper decoupler

$$G_I(s) = \hat{G}^{T}(s) G_R(s)$$

$$= \begin{bmatrix} -12.4533e^{-5.9666s} & \frac{423.3268s+4.5269}{121.3735s+1}e^{-5.4125s} & \frac{73.0789s+0.8838}{113.9022s+1}e^{-5.4535s} & \frac{112.6970s+1.1876}{123.5480s+1}e^{-3.6602s} \\ \frac{532.6803s+5.6980}{113.8299s+1}e^{-4.8878s} & -13.2626e^{-6.3777s} & \frac{52.2751s+0.5300}{113.9022s+1} & \frac{49.1501s+0.4144}{123.5480s+1} \\ \frac{40.4368s+0.3752}{113.8299s+1} & \frac{110.4244s+1.2350}{121.3735s+1}e^{-2.4022s} & -10.8814e^{-6.7631s} & \frac{296.1062s+3.1279}{123.5480s+1}e^{-6.6871s} \\ \frac{60.8111s+0.6072}{113.8299s+1}e^{-2.4454s} & \frac{94.9821s+0.9650}{121.3735s+1} & \frac{264.3081s+2.7412}{113.9022s+1}e^{-5.0869s} & -10.2987e^{-5.7511s} \end{bmatrix} \quad ...(34)$$

By selecting a gain margin of 5db and a phase margin of 2®/5, the diagonal controller, $G_C(s)$, is designed as

$$G_C(s)=\begin{bmatrix} 1.639+\frac{0.0144}{s} & 0 & 0 & 0 \\ 0 & 1.789+\frac{0.01474}{s} & 0 & 0 \\ 0 & 0 & 1.611+\frac{0.01415}{s} & 0 \\ 0 & 0 & 0 & 1.6784+\frac{0.0136}{s} \end{bmatrix} \quad ...(35)$$

Thus, the final decoupling controller is obtained as

$$C(s) = G_I(s)G_C(s)$$

$$= \begin{bmatrix} \frac{-20.4110s-0.1793}{s}e^{-5.966s} & \frac{757.3316s^2+14.4711s+0.0667}{121.3735s^2+s}e^{-5.415s} & \frac{117.7301s^2+2.4579s+0.0125}{113.9022s^2+s}e^{-5.453s} & \frac{189.1506s^2+3.5259s+0.01615}{123.5480s^2+s}e^{-3.602s} \\ \frac{873.0630s^2+17.0096s+0.0821}{113.8299s^2+s}e^{-4.878s} & \frac{-23.7268s-0.1955}{s}e^{-6.377s} & \frac{842.152s^2+1.5935s+0.0075}{113.9022s^2+s} & \frac{82.4935s^2+1.3640s+0.0056}{123.5480s^2+s} \\ \frac{66.2759s^2+1.1972s+0.0054}{113.8299s^2+s} & \frac{119.5492s^2+3.8371s+0.0182}{121.3735s^2+s}e^{-2.402s} & \frac{-17.5299s-0.1540}{s}e^{-6.761s} & \frac{496.9846s^2+9.2769s+0.1425}{123.5480s^2+s}e^{-6.871s} \\ \frac{99.6694s^2+1.8709s+0.00874}{113.8299s^2+s}e^{-2.454s} & \frac{169.9230s^2+3.1264s+0.01820}{121.3735s^2+s} & \frac{425.8003s^2+1.5935s+0.0388}{113.9022s^2+s}e^{-5.036s} & \frac{-17.2853s-0.1401}{s}e^{-5.751s} \end{bmatrix} \quad ...(36)$$

SIMULATION RESULTS

Figure 2 shows the Simulink diagram of the 4 x 4 Decoupled system with the servo and regulatory inputs. Figure 3 shows the output response of the servo and regulatory responses with various inputs as 2, 1.5, 1 and 0.5 in the step inputs and with the disturbance magnitude of 0.1 at 7000 sample.

Figure 2: Simulink Diagram of MIMO with Decoupler

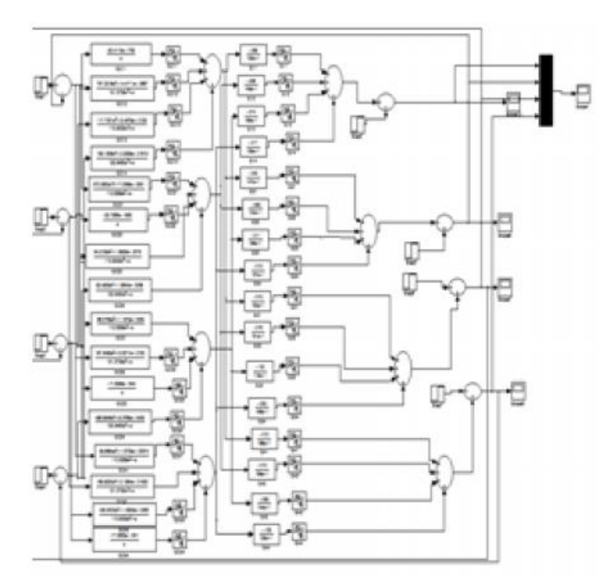
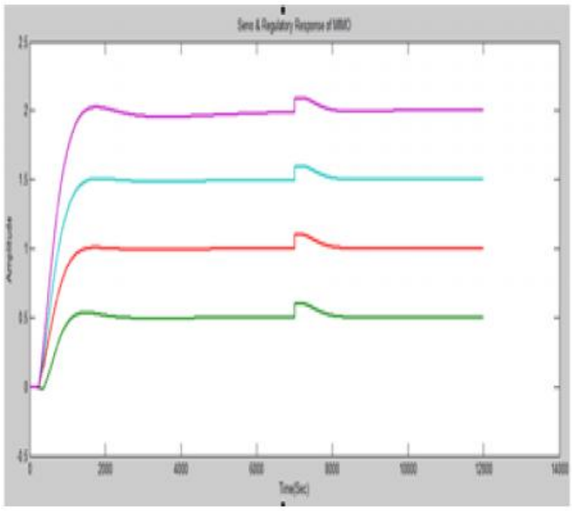


Figure 3: Servo and Regulatory Response of MIMO



CONCLUSION

In this paper we have designed and implemented the RGA based decoupler for the 4 x 4 MIMO process and obtained the satisfactory servo and regulatory responses as shown in Figure 4. Further the design of Model Reference Adaptive Controller needs to be designed for the MIMO process to get the top and bottom product with the specified purity rate.

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