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Research Paper

A COMPARATIVE STUDY OF WAVELET TRANSFORM TECHINQUE AND SVD IN THE ESTIMATION OF POWER SYSTEM HARMONICS AND INTERHARMONICS

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This paper presents a comparison of Wavelet Transform technique and Singular Value Decomposition technique in the estimation of power system harmonics and interharmonics. Singular Value Decomposition technique based method has been used to estimate the harmonics and Interharmonics by calculating the amplitudes of the frequencies under consideration. Now-a-days, Wavelet Transformation is one of the most popular candidates of time-frequency transformation. Because Wavelet Transformation can provide time and frequency information simultaneously and it is suitable for the analysis of non-stationary signal. To investigate these methods, a number of studies have been performed using simulated signals. The analysis of the voltage waveform of a PWM converter supplying an Induction motor has been investigated employing these two methods with the same sampling period.

Keywords: CWT, Harmonics, Interharmonics, SVD

INTRODUCTION

An ideal power system is defined as the system where a perfect sinusoidal voltage signal is seen at load-ends. In reality, however, such idealism is hard to maintain (Arumugam *et al.*, 2011). It is because, the widespread applications of electronically controlled loads have increased the harmonic distortion in power system voltage and current waveforms. As power semiconductors are switched on and off at different points on the voltage waveform, damped high-frequency transients are generated. If the switching occurs at the same points on each cycle, the transient becomes periodic (Adley Girgis *et al.*, 1991). Harmonic frequencies in the power grid are a frequent cause of power quality problems. Harmonics in power systems result in increased heating in the equipment and conductors, misfiring in variable speed drives, and torque pulsations

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in motors. So, estimation and reduction of harmonics is very important. Many algorithms have been proposed for the evaluation of harmonics. The design of harmonic filters relies on the measurement of harmonic distortion (Adley Girgis *et al.*, 1991). Harmonics State Estimation (HSE) techniques have been used since 1989 for harmonics analysis in power systems. Many mathematical methods have been developed over the years. It is proved that by using only partial or selected measurement data, the entire harmonic distortion of the actual power system can be obtained effectively. In this paper, the performances of Singular Value Decomposition (SVD) and Wavelet Transform technique have been compared in estimation of power system harmonics.

In WT method, Continuous Wavelet Transform (CWT) is applied to the signal. The Morlet wavelet is applied as the mother wavelet to estimate the frequencies of the signal. It is suitable for the analysis of non-stationary signal.

Singular Value Decomposition (SVD) technique is a higly reliable, computationally stable mathematical tool to solve the rectangular overdetermined system of equations (Osowski, 1994). By solving the equations, the amplitudes of the frequencies present in the signal are determined.

The principles of these two methods are explained in 2nd and 3rd sections, the experimental results are given in 4th and 5th sections respectively and conclusion is given in the final section.

WAVELET TRANSFORM TECHNIQUE

In this approach the signal is subjected Continuous Wavelet Transform to estimate the harmonics and interharmonics.

Continuous Wavelet Transform

The CWT of a continuous, square-integrable function $x(t)$ at scale $a > 0$ and translational value b € R is expressed by the following integral-

$$CWT(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) \Psi^* \left(\frac{t-b}{a} \right) dt \qquad ...(1)$$

where $\frac{1}{\sqrt{|a|}}$ is the normalization factor, $\Psi(t)$ is called mother wavelet which is a continuous function both in time domain and frequency domain. The main purpose of the mother wavelet is to provide a source function to generate the daughter wavelets which are simply the translated and scaled version of mother wavelet.

Harmonics and Interharmonics Estimation

To estimate the harmonics and interharmonics, CWT is applied to the signal. The Morlet wavelet is selected to be the mother wavelet. It is defined in time domain as follows (Keaochantranond and Boonseng, 2002):

$$\cdot\ (t) = \exp\left(j\mu_{ow} t - 0.5 t^2\right) \qquad ...(2)$$

where μ_{ow} = 2®f_{ow};f_{ow} is frequency of Morlet wavelet. The relationship between scale and frequency in CWT is given by:

$$f_a = \frac{f_{ow}}{a\Delta} \qquad ...(3)$$

where a = scale, f_a = frequency corresponding to the scale a, Δ = sample. The table showing the scales and their corresponding frequencies is first determined and then the scalograms are obtained for the signal at different scales for the estimation. The maximum energy points represent the scales corresponding to the frequencies present in the signals. Order of harmonics and interharmonics can be found from the following expression as (Keaochantranond and Boonseng, 2002):

$$Order\ of\ harmonics = \frac{Harmonic\ frequency}{System\ frequency} \quad ...(4)$$

SINGULAR VALUE DECOMPOSITION TECHNIQUE

Let the waveform of the voltage or current be represented as the sum of harmonics of unknown magnitudes and phases as:

$$x(t) = \sum_{k=1}^{N} X_k \sin\{\mu kt + ¤_k\} \quad ...(5)$$

or,

$$x(t) = \sum_{k=1}^{N} \{A_k \sin(\mu kt) + B_k \cos(\mu kt)\} \quad ...(6)$$

where X_k = unknown amplitude of k^{th} harmonic, μ = fundamental angular frequency, $¤_k$ = phase of the k^{th} harmonic, N = total no. of harmonics.

From Equations (5) and (6), the following relations are obtained:

$$X_k = \sqrt{A_k^2 + B_k^2} \quad ...(7)$$

$$¤_k = \arctan\left(\frac{B_k}{A_k}\right) \quad ...(8)$$

Now, let us consider n measured samples $x_1, x_2, \ldots, x_n$ of the waveform. The number of measurements are usually higher than the number of harmonics. Estimation of harmonics is equivalent to solving an overdetermined system of algebraic equations expressed as (Osowski, 1994):

$$Ah = b \quad ...(9)$$

where

$$b = [x_1, x_2, x_3, \ldots, x_n]^t$$

$$h = [A_1, B_1, A_2, B_2, \ldots, A_N, B_N]^t \quad ...(10)$$

So, h is a vector of unknown components A_k, B_k of harmonics under consideration. A is an n x 2N matrix correlating vectors h and b as shown below.

$$A = \begin{bmatrix} \sin\mu t_1 \ \cos\mu t_1 & \sin 2\mu t_1 \ \cos 2\mu t_1 & \cdots & \sin N\mu t_1 \ \cos N\mu t_1 \\ \sin\mu t_2 \ \cos\mu t_2 & \sin 2\mu t_2 \ \cos 2\mu t_2 & \cdots & \sin N\mu t_2 \ \cos N\mu t_2 \\ \cdots & \cdots & \cdots & \cdots \\ \sin\mu t_n \ \cos\mu t_n & \sin 2\mu t_n \ \cos 2\mu t_n & \cdots & \sin N\mu t_n \ \cos N\mu t_n \end{bmatrix} \quad ...(11)$$

For solution of vector h, the most suitable offline method is SVD (IEEE Working Group on Power System Harmonics, 1983; and Osowski, 1994). In this method the rectangular n x 2N matrix A is represented as the product of three matrices:

$$A = USV^t \quad ...(12)$$

where U and V are orthogonal matrices of the dimensions n x n and 2N x 2N respectively. S is quasidiagonal n x 2N matrix of singular values S_1, S_2, ..., S_p ordered in a descending way. The essential information of the system is contained in the first nonzero singular values and first p singular vectors, forming the orthogonal matrices U and V. Reducing the appropriate matrices to this size and denoting them by U_r, S_r and V_r respectively, the solution of (9) will lead to

$$h = V_r S_r^{-1} U_r^t b \quad ...(13)$$

The reduced size matrices, U_r and V_r are created from the original matrices by taking the first p columns from the matrix U and the first p rows from the matrix V, respectively. The diagonal matrix S_r, is formed from the original matrix S by the nonzero diagonal entries, hence

$$S_r^{-1} = diag\left[\frac{1}{S_1}, \frac{1}{S_2}, \cdots, \frac{1}{S_p}\right]$$

As seen from Equation (13), the only operations that should be performed to solve the overdetermined system of eqns. 9, by using SVD, is the multiplication of the appropriate reduced-size matrices. If matrix C is constant, then SVD is performed only once and the solution in the form of eqn. 13 may be repeated many times at the presentation of different measurement vectors *d* (Osowski, 1994). The values of vector h provide the amplitudes of harmonics and Interharmonics present in the signal and thus estimate the frequencies present in the signal. A prior knowledge of the power system under consideration is needed to estimate the frequencies by using SVD.

EXPERIMENTS WITH SIMULATED WAVEFORM

The first signal considered is given by:

$$x(t) = 100\cos(2®40t) + 50\cos(2®217t) + 40\cos(2®760t) + noise \quad ...(14)$$

The signal is corrupted with a white Gaussian noise of zero mean and variance equal to 1. The Signal to Noise Ratio (SNR) is 10. To investigate the methods, several experiments have been performed with the waveform described by (14) and given by Figure 1. Sampling frequency is taken as 2000 Hz for both the methods. The number of samples is taken as 2000.

Wavelet Transform

The CWT is applied to the signal with Morlet as the mother wavelet and with the sampling

Figure 1: First Signal

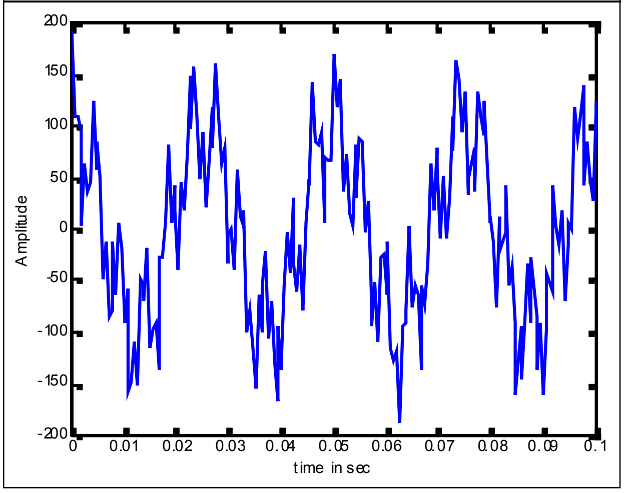


Figure 2: Scale vs Frequency Curve for Morlet Wavelet with Sampling Frequency of 2000 Hz

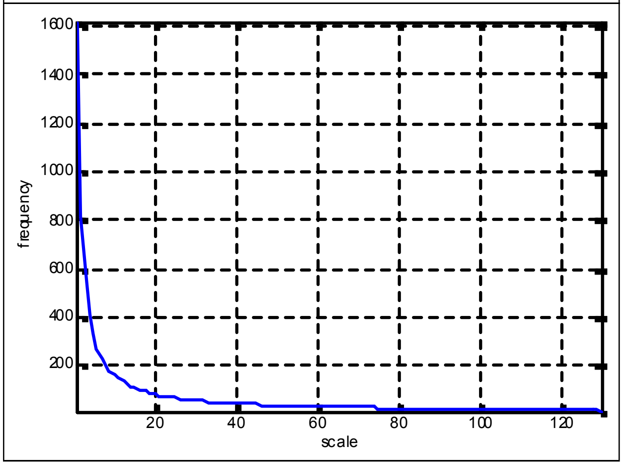


Figure 3: Fundamental Frequency Estimated as 40.625 Hz at Scale 40 by WT

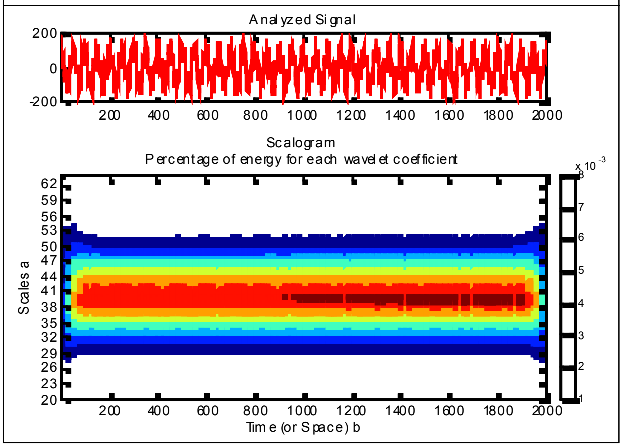


Figure 4: Interharmonic Estimated as 216.67 Hz at Scale 7.5

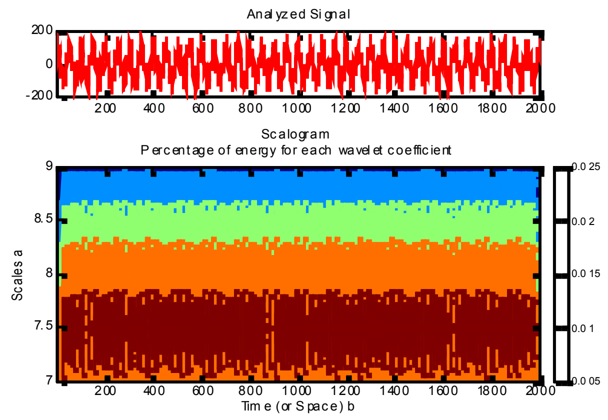
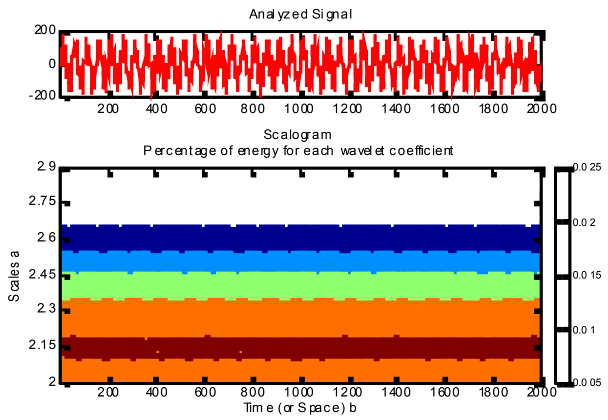


Figure 5: 19th Harmonic Estimated as 755.81 Hz at Scale 2.15



frequency of 2000 Hz. The scale vs frequency curve and the highest energy points corresponding to the estimated frequencies are shown below.

Singular Value Cecomposition

For the signal given by Equation (14) , SVD is applied to estimate the frequencies with sampling frequency 2000 Hz. The number of samples taken for this analysis is 2000. The fundamental frequency 40 Hz , Interharmonic 217 Hz and 19th harmonic 760 Hz present in the signal are taken for consideration to constitute the matrix A. The frequencies are estimated as shown in the following table.

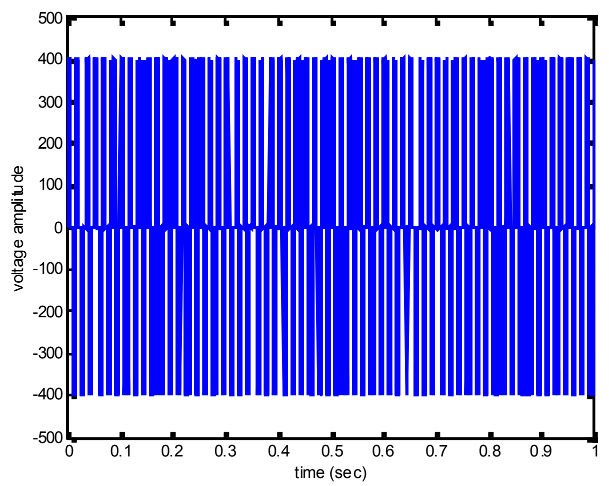
Table 1: Amplitudes of the Frequencies Present in the First Signal Estimated by SVD

Frequency (Hz)	Amplitude (X_k)
40	99.9904
217	50.0128
760	40.0070

SIMULATION OF A FREQUENCY CONVERTER

A PWM converter with modulation frequency of 1080 Hz supplying a 4 pole, 3 hp asynchronous motor (U = 220 V) is simulated in simulink. The simulated converter has a modulation index of 0.92. The output voltage waveform of the converter corrupted with noise having zero mean value and unity variance is taken for analysis. Figure 6 shows the noise corrupted voltage waveform at the converter output for the frequency 60 Hz.

Figure 6: Simulated Voltage Waveform



Wavelet Transform

Continuous Wavelet Transform is applied to the voltage signal with sampling frequency 6400 Hz. Again, Morlet wavelet is considered as mother wavelet. The first 2048 samples are taken for this analysis. The scale Vs frequency

Figure 7: Frequency vs Scale Curve for Morlet Wavelet at Sampling Frequency 6400 Hz

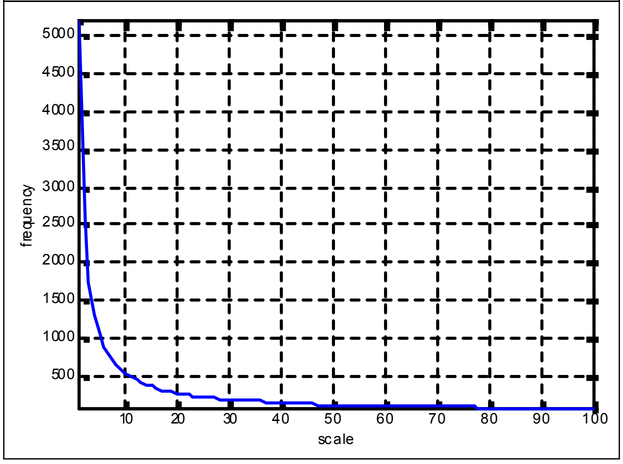


Figure 8: The Fundamental Frequency Estimated as 59.77 Hz at Scale 87

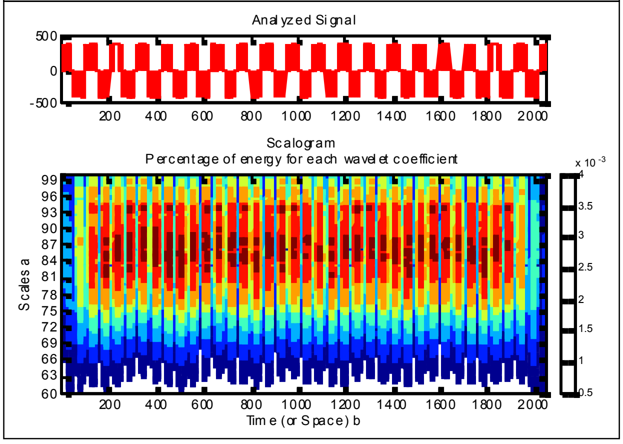


Figure 9: Frequency of 1040.2 Hz Estimated at Scale 5

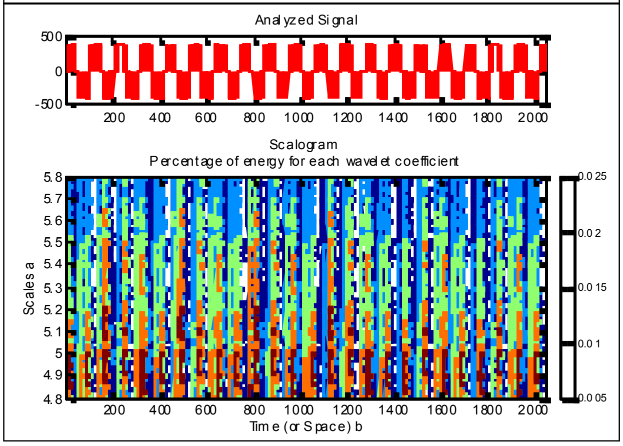


Figure 10: Frequency of 2131.5 Hz Estimated at Scale 2.44

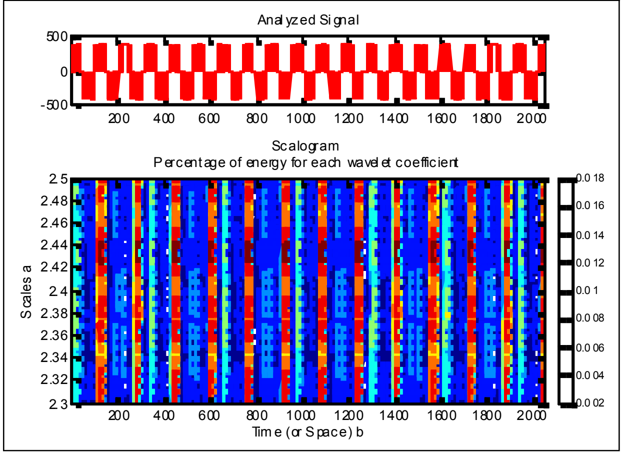


Figure 11: Frequency of 2241.7 Hz Estimated at Scale 2.32

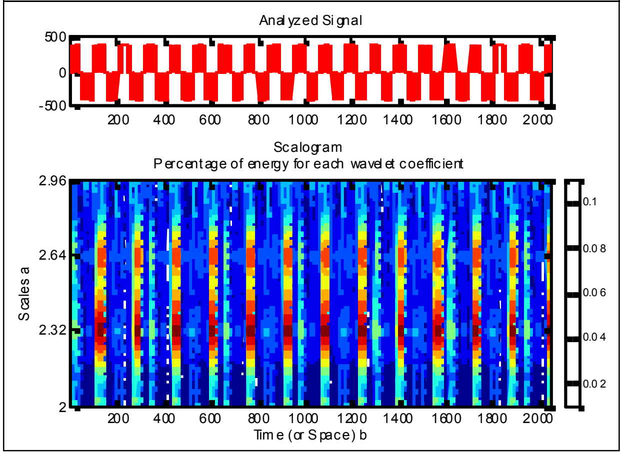


Figure 12: Frequency of 2873.4 Hz Estimated at Scale 1.81

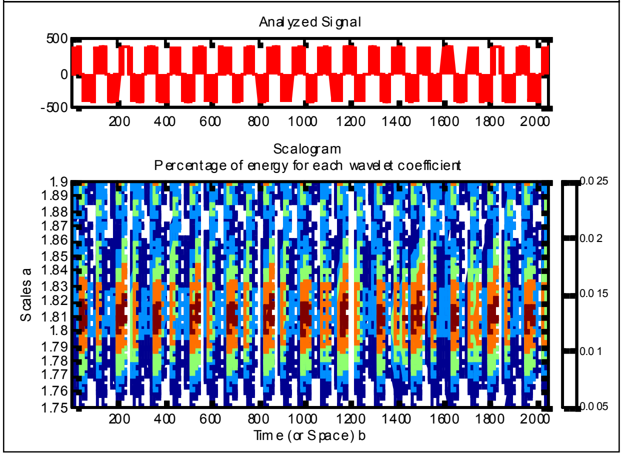
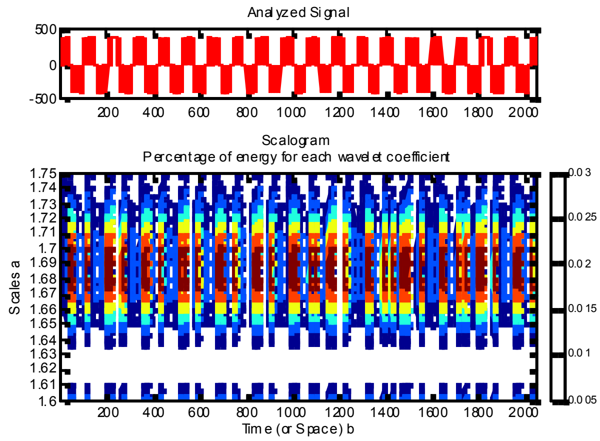


Figure 13: Frequency of 3077.4 Hz Estimated at Scale 1.69



curve for Morlet wavelet at sampling frequency 6400 Hz is shown in Figure 7. The major frequencies estimated by WT are shown in the following figures from Figures 8 to 13.

Wavelet Transform estimates the major frequencies as: 59.77, 1040.2, 2031.6, 2072, 2131.5, 2241.7, 2667.1, 2873.4, 3077.4, 3333.9 Hz.

SVD

SVD is applied to the simulated voltage signal to estimate the frequencies present in the signals as explained in previous section. The sampling frequency is 6400 Hz and first 2048 samples are taken for the analysis. A prior knowledge of the system under consideration is required to apply SVD technique. The amplitudes of the major harmonics and interarmonics estimated by SVD are obtained as shown in Table 2.

Table 2: Amplitudes Corresponding to the Frequencies Estimated for the Simulated Voltage Signal

Frequency (Hz)	Amplitude (X_k)
60	321.9131
960	99.1468
1200	98.9905
2100	77.8872
2220	78.0127
3000	50.6782
3030	39.9028
3120	72.8911

CONCLUSION

The estimation of harmonics and interharmonics in a power system has been investigated using SVD and Wavelet Transform for different test signals with same no. of samples and sampling period. It is observed that Wavelet Transform technique is not as accurate as the Singular Value Decomposition technique in the estimation of frequencies in case of stationary signal. However, wavelets, though not specifically dedicated to this type of analysis, can recover some of the spectral information. In case of non-stationary signal, Wavelet analysis can estimate the frequencies as well as can detect time of occurrence of the frequencies. SVD is an efficient tool in estimating amplitudes of harmonics and interharmonics existing in a system. Although SVD requires a prior knowledge of the signals under consideration and because of significant computational work involved, SVD is mostly suitable for the offline analysis of recorded waveform.

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