Another Look at the VSI EWMA $\bar{X}$ Chart when Estimating Process Parameters

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***Abstract*—The performance of the variable sampling interval (VSI) exponentially weighted moving average (EWMA) chart is generally investigated under the assumption of known process parameters. Nevertheless, the process parameters need to be estimated from a historical Phase-I dataset because they are usually unknown in practice. When the process parameters are estimated, the chart's performance differs among practitioners as different number of Phase-I samples is used. This leads to different parameter estimates in constructing the chart's limits and variation in the average time to signal (ATS). This type of variation is crucial to be considered when evaluating the performance of the control chart with estimated process parameters. To consider practitioner-to-practitioner variation, this paper investigates the performance of the VSI EWMA $\bar{X}$ chart with estimated process parameters by using standard deviation of the ATS. Monte Carlo simulation results show that the VSI EWMA $\bar{X}$ chart requires many Phase-I samples to achieve the desired performance. The results also show that a greater number of Phase-I samples is needed for the VSI EWMA $\bar{X}$ chart when the smoothing constants are large. This is because larger values of smoothing constants lead to higher variation in the run-length distribution.**

***Index Terms*—EWMA control chart, expected value of the average time to signal, parameter estimation, standard deviation of the average time to signal, statistical process control, variable sampling interval**

I. Introduction

Control charts are extensively used for maintaining the consistency of a manufacturing process in an industry. They are known to be the most useful tool in Statistical Process Control. A well-designed control chart enables practitioners to detect shifts or any presence of assignable causes in manufacturing processes before defects occur. The Exponentially Weighted Moving Average (EWMA) control chart considers the weighted average of all current and past observations. This feature gives the EWMA chart with the advantage of being more sensitive in detecting small and moderate shifts. To improve the inspection and statistical efficiency of the EWMA chart, an adaptive strategy such as variable sample interval (VSI) is adopted to design the EWMA chart. Therefore, when a process shift occurs, an out-of-control signal is obtained more quickly by varying the sampling interval of the EWMA chart, compared to that of the fixed-sampling-interval EWMA chart.

The VSI EWMA $\bar{X}$ chart was first proposed by [1]. They studied the run-length properties of the VSI EWMA $\bar{X}$ chart with known process parameters by means of the Markov chain approach. Castagliola, Celano, and Fichera [2] studied the statistical performance of the VSI *R* EWMA control chart to monitor the range. Castagliola, Celano, Fichera, and Giuffrida [3] proposed the VSI S^2-EWMA chart to monitor process variance. To reduce the cost in process production cycle, economic models of the VSI EWMA control chart were proposed by [4] and [5] under normality and non-normality assumptions, respectively. Lin and Chou [6] investigated the VSI EWMA and combined VSI $\bar{X}$ –EWMA charts when the normality assumption of the observed data or measurements is violated. Furthermore, the run-length properties of the multivariate VSI EWMA chart were evaluated in Lee and Khoo [7]. Liu, Chen, Zhang, and Zi [8] presented the VSI nonparametric EWMA chart, which is a distribution-free control chart. Recently, the one-sided EWMA *t* charts with and without variable sampling intervals for monitoring the process mean was proposed by [9]. They found that the VSI one-sided EWMA *t* chart is more efficient than the corresponding chart without VSI feature, in detecting different shift sizes. By applying the joint economic model, the VSI EWMA control charts are found to be more effective in reducing loss [10]. To

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enhance the accuracy of the VSI EWMA chart, [11] adopted the auxiliary-information based scheme into the VSI EWMA chart. Substantially, all works on developing the VSI EWMA-type control charts enhance the statistical efficiency of the control charts compared to their corresponding EWMA-type charts with fixed sampling interval.

There are two phases of process monitoring, namely Phase-I and Phase-II. An in-control dataset will be collected in Phase-I for the purpose of estimating process parameters and constructing control limits used in Phase-II. In Phase-II, control charts involve inspection of future production. The main purpose of Phase-II control charts is to detect process shifts efficiently when the process moves from in-control to out-of-control condition. Usually, the performance of a control chart with estimated process parameters is evaluated in terms of the average time to signal (ATS) [12]-[14]. When the process parameters are estimated in Phase-I, the ATS becomes a random variable. This randomness of the chart's performance among practitioners is expected. This type of randomness is called practitioner-to-practitioner variation. This variation is due to different practitioners adopt different number of Phase-I data. Consequently, the estimation of the process parameters differs from one practitioner to another, leading to different ATS values. Recently, [15] reviewed the effect of process parameter estimation on Shewhart, EWMA, and CUSUM control charts. They also stated that even when the actual distribution of the data is known, this practitioner-to-practitioner variation is small only for very large numbers of Phase-I data. In order to solve this problem, one of the proposed methods is to use bootstrap method to adjust the control limits based on the conditional perspective [16], [17]. Recently, [18] and [19] showed that under the conditional perspective, the performances of the Shewhart $\bar{X}$ and S^2 control charts, respectively, with estimated process parameters using the exact, approximate, and bootstrap methods to adjust the control limits, yield similar results.

The standard deviation of the ATS (SDATS) accounts for practitioner-to-practitioner variability. This SDATS metric is more suitable in accessing the performance of the control charts with estimated process parameters. Therefore, in this paper, the performance of the VSI EWMA $\bar{X}$ chart when the process parameters are estimated, is evaluated with the expected value of the ATS (AATS) and SDATS metrics. A similar metric, i.e. standard deviation of the average run length (SDARL) was proposed by Jones and Steiner [20] to evaluate the performance of the risk adjusted CUSUM chart with estimated process parameters. This metric is further used by other authors in evaluating the performance of different type of control charts when the process parameters are estimated [21]-[24]. Note that the aim of this paper is to consider the practitioner-to-practitioner variation in preliminary investigating the effects of the VSI EWMA $\bar{X}$ chart when the process parameters are estimated.

The remainder of this paper is organized as follows. In Section II, we present a brief overview of the VSI EWMA $\bar{X}$ chart. By means of the Monte Carlo simulation, the results of the AATS and SDATS of the VSI EWMA $\bar{X}$ chart with estimated process parameters, are provided in Section III. Finally, we provide some concluding remarks in Section IV.

II. The VSI EWMA $\bar{X}$ Chart

Assume that ($Y_{i,1}, Y_{i,2}, \cdots, Y_{i,n}$) is a sample taken from Phase-II process. Here, i = 1, 2, ⋯, is the subgroup number. This ($Y_{i,1}, Y_{i,2}, \cdots, Y_{i,n}$) sample consists of n independent normal random variables with in-control mean, μ_0 and in-control variance, σ_0^2. The VSI EWMA $\bar{X}$ chart is divided into three regions, which are the safe region, warning region and out-of-control region as shown in Fig. 1.

Let K_1 (> 0) and K_2 (> K_1) be the warning limit coefficient and control limit coefficient, respectively. The upper (UCL) and lower (LCL) control limits, as well as the upper (UWL) and lower (LWL) warning limits of the VSI EWMA $\bar{X}$ control chart are computed as follows:

$$\text{UCL} / \text{LCL} = \pm K_2 \sqrt{\frac{\lambda}{2-\lambda}} \quad (1)$$

and

$$\text{UWL} / \text{LWL} = \pm K_1 \sqrt{\frac{\lambda}{2-\lambda}}\,, \quad (2)$$

respectively, where $\lambda \in (0,1]$ is the smoothing constant. By referring to Fig. 1, the procedure for implementing the VSI EWMA $\bar{X}$ chart is as follows:

1. Collect a random sample of n (> 1) observations.
2. Specify the values of the chart's parameters λ, K_1 and K_2, in order to compute the control limits and warning limits as in (1) and (2), respectively.
3. Calculate the standardized sample mean, $W_i = (\bar{Y}_i - \mu_0)/(\sigma_0/\sqrt{n})$ of subgroup i and the VSI EWMA $\bar{X}$ chart's statistic, $Z_i = \lambda W_i + (1-\lambda) Z_{i-1}$, where i = 1, 2, ...

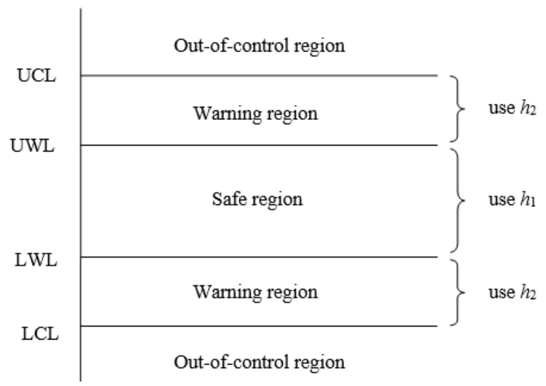

Fig. 1. Graphical view of the VSI EWMA $\bar{X}$ chart's operation.

4. The process is declared as in-control when $Z_i \in$[LWL, UWL], then a long sampling interval, h_1 is used for next sampling.
5. The process is also declared as in-control when $Z_i \in$[UWL, UCL] or $Z_i \in$[LCL, LWL]. In this case, a short sampling interval, h_2 is used for next sampling.
6. The process is declared as out-of-control when $Z_i \notin [\text{LCL}, \text{UCL}]$. Then corrective actions are taken to investigate and omit the assignable cause(s).

To evaluate the performance of the VSI EWMA $\bar{X}$ chart with known process parameters, the Markov Chain approach is adopted to model its run-length properties. This approach involves the operation of dividing the interval between UCL and LCL into $2g + 1$ subintervals, each of width $2d$, where $2d$ = (UCL – LCL)/($2g + 1$). In our case, the $(2g + 1) \times (2g + 1)$ matrix **R** of transient probabilities [25] is equal to

$$\mathbf{R} = \begin{pmatrix} R_{-g,-g} & \cdots & R_{-g,-1} & R_{-g,0} & R_{-g,+1} & \cdots & R_{-g,+g} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{-1,-g} & \cdots & R_{-1,-1} & R_{-1,0} & R_{-1,+1} & \cdots & R_{-1,+g} \\ R_{0,-g} & \cdots & R_{0,-1} & R_{0,0} & R_{0,+1} & \cdots & R_{0,+g} \\ R_{+1,-g} & \cdots & R_{+1,-1} & R_{+1,0} & R_{+1,+1} & \cdots & R_{+1,+g} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{+g,-g} & \cdots & R_{+g,-1} & R_{+g,0} & R_{+g,+1} & \cdots & R_{+g,+g} \end{pmatrix} \quad (3)$$

where the generic elements $R_{k,\ell}$ of matrix **R** are equal to

$$R_{k,\ell} = \Phi\left(\frac{H_\ell + d - (1-\lambda)H_k}{\lambda} - \delta\sqrt{n}\right) - \Phi\left(\frac{H_\ell - d - (1-\lambda)H_k}{\lambda} - \delta\sqrt{n}\right), \quad (4)$$

with k, $\ell = -g, \cdots, -1, 0, 1, \cdots, g$. Here, H_k represents the midpoint of the k^{th} subinterval. The magnitude of the standardized mean shift is $\delta = |\mu_1 - \mu_0| / \sigma_0$, where μ_1 is the out-of-control mean. The standard normal cumulative distribution function (cdf) is represented by $\Phi(\cdot)$. As shown in [1], the ATS of the VSI EWMA $\bar{X}$ chart is computed as follow:

$$\text{ATS} = \mathbf{q}^{\text{T}}\mathbf{Q}\mathbf{b} - \mathbf{q}^{\text{T}}\mathbf{b}, \quad (5)$$

where the fundamental matrix is $\mathbf{Q} = (\mathbf{I} - \mathbf{R})^{-1}$, the initial probability vector is $\mathbf{q} = (0, \cdots, 1, \cdots, 0)^T$, the vector of sampling intervals corresponding to the discretized states of Markov chain is represented by **b**, and the identity matrix is **I**.

When both the process parameters, i.e. the in-control mean, μ_0 and the in-control standard deviation, σ_0 are unknown, they are necessary to be estimated from m in-control Phase-I samples, each of n observations, i.e. $\{X_{i,1}, X_{i,2}, \cdots, X_{i,n}\}$, where $i = 1, 2, \cdots, m$. The parameter μ_0 is estimated by

$$\hat{\mu}_0 = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} X_{i,j}\ ; \quad (6)$$

while the parameter σ_0 is estimated by

$$\hat{\sigma}_0 = \frac{S_{\text{pooled}}}{c_{4,m}}, \quad (7)$$

where

$$S_{\text{pooled}} = \sqrt{\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(X_{i,j} - \bar{X}_i\right)^2}{m(n-1)}} \quad (8)$$

$$c_{4,m} = \frac{\sqrt{2}\,\Gamma\left[\left(m(n-1)+1\right)/2\right]}{\sqrt{m(n-1)}\,\Gamma\left[\left(m(n-1)\right)/2\right]}. \quad (9)$$

Here, $\Gamma(\cdot)$ is a gamma function.

Each practitioner gets different estimates because different number of Phase-I samples is adopted. Therefore, sampling variation is necessary to be considered in the case when process parameters are estimated. Usually, the performance of control charts with estimated process parameters is assessed with AATS. However, the metric AATS does not reflect the variability between practitioners. On the other hand, the SDATS is used to measure the between-practitioner variability. Therefore, it is necessary to consider the SDATS in evaluating the performance of the control charts with estimated process parameters. In this paper, both the AATS and SDATS metrics are used to evaluate the impact of the estimation error on the performance of the VSI EWMA $\bar{X}$ chart with estimated process parameters. We recommend that the SDATS value should be within 5% to 10% of the desired ATS value in order to achieve reasonable charts' performance. Similar approach is also suggested by [21], [26], and [27].

III. A Study of the Monte Carlo Simulation

In this section, Monte Carlo simulation is used to obtain the in-control AATS (AATS_0) and SDATS (SDATS_0), as well as the out-of-control AATS (AATS_1) and SDATS (SDATS_1) of the VSI EWMA $\bar{X}$ chart with estimated process parameters. This simulation program is written by using the Statistical Analysis System (SAS). A total of 100,000 simulation runs are applied in this simulation study. The process mean, $\hat{\mu}_0$ and process standard deviation, $\hat{\sigma}_0$ are estimated in our simulation study by using the estimators in (6) and (7), respectively. We consider different values of m, ranging from 200 to 4000, with a fixed sample size $n = 5$. Three values of $\lambda \in \{0.1, 0.2, 0.5\}$ and three $(h_1, h_2) \in \{(1.5, 0.5), (1.7, 0.3), (1.3, 0.1)\}$ combinations are employed in this paper. For each combination of (λ, h_1, h_2), the values of K_1 and K_2 are calculated by using the formulae shown in Section II, subject to the constraint $\text{ATS}_0 \in \{370, 500\}$. Note that the ATS measurement is used in the case with known process parameters; while the AATS measurement is used in the case with estimated process parameters.

A. In-Control Performances

Table I to Table III display the $AATS_0$ and $SDATS_0$ values for $\lambda \in \{0.1, 0.2, 0.5\}$, respectively, $ATS_0 \in \{370, 500\}$, n = 5, different values of m and different combinations of (h_1, h_2, K_1, K_2). The last row of each table, i.e. $m = +\infty$ represents the case when the process parameters are known. Note that in Tables I to III, the boldfaced $AATS_0$ values are about 98% of the desired ATS_0 values; while the boldfaced $SDATS_0$ values are about 10% of the desired ATS_0 values. When the process parameters are known ($m = +\infty$), the $SDATS_0$ and $SDATS_1$ are both equal to zero.

It is clear from Table I to Table III that the $AATS_0$ values increase and approach the desired ATS_0 values as m increases. This is because as m increases, the variability in the sampling distribution of the estimators decreases. The results also show that smaller m is needed to achieve $AATS_0$ of 98% of the desired ATS_0 when larger value of λ is used. For example, when (h_1, h_2) = (1.5, 0.5), ATS_0 = 370, and n = 5, the $AATS_0$ for the VSI EWMA $\bar{X}$ chart with λ = 0.1 requires m = 1800 (see Table I) to achieve 98% of the ATS_0 = 370; however, this m decreases to m = 950 (see Table II) when λ = 0.2. It is further decreasing to m=250 (see Table III) when λ =0.5.

TABLE I: $AATS_0$ AND $SDATS_0$ VALUES FOR λ = 0.1, $ATS_0 \in \{370, 500\}$, AND DIFFERENT COMBINATIONS OF (h_1, h_2, K_1, K_2), WHEN DIFFERENT NUMBER OF PHASE-I SAMPLES m, EACH HAVING n = 5 OBSERVATIONS, ARE USED TO ESTIMATE THE IN-CONTROL PHASE-I PROCESS PARAMETERS

ATS_0 = 370						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
K_1	0.6269		0.6312		1.0916	
K_2	2.7067		2.7085		2.7051	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
500	343.39	54.04	343.04	55.15	343.00	54.79
800	352.71	41.43	352.57	42.28	352.47	42.00
1000	356.14	**36.41**	356.07	**37.16**	355.95	**36.91**
1100	357.44	34.45	357.40	35.16	357.27	34.93
1200	358.53	32.75	358.52	33.43	358.38	33.20
1300	359.48	31.27	359.48	31.91	359.34	31.69
1400	360.30	29.95	360.32	30.56	360.17	30.36
1500	361.02	28.77	361.06	29.36	360.90	29.17
1600	361.66	27.72	361.71	28.29	361.55	28.10
1700	362.22	26.76	362.29	27.31	362.12	27.13
1800	**362.73**	25.89	**362.81**	26.42	**362.64**	26.25
1900	363.19	25.10	363.28	25.61	363.10	25.44
2000	363.61	24.37	363.70	24.87	363.53	24.70
2100	363.98	23.70	364.09	24.18	363.91	24.02
2200	364.33	23.07	364.44	23.54	364.26	23.39
+∞	370.00	0	370.00	0	370.00	0
ATS_0 = 500						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
K_1	0.6485		0.6591		1.1000	
K_2	2.8145		2.8141		2.8166	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
500	460.29	78.76	459.43	80.10	459.70	80.00
800	473.88	60.57	473.27	61.602	473.52	61.53
1000	478.92	53.29	478.39	54.20	478.64	54.13
1100	480.83	**50.44**	480.33	**51.30**	480.58	**51.23**
1200	482.45	47.96	481.98	48.78	482.22	48.72
1300	483.84	45.79	483.40	46.57	483.64	46.51
1400	485.05	43.87	484.64	44.61	484.87	44.56
1500	486.12	42.15	485.72	42.87	485.96	42.81
1600	487.06	40.60	486.68	41.30	486.92	41.24
1700	487.90	39.21	487.54	39.87	487.78	39.82
1800	488.66	37.93	488.31	38.58	488.54	38.53
1900	489.34	36.77	489.00	37.40	489.23	37.35
2000	489.96	35.70	489.63	36.31	489.86	36.26
2100	**490.52**	34.71	**490.20**	35.30	**490.43**	35.26
2200	491.03	33.80	490.73	34.37	490.96	34.33
+∞	500.00	0	500.00	0	500.00	0

TABLE II: $AATS_0$ AND $SDATS_0$ VALUES FOR λ = 0.2, $ATS_0 \in \{370, 500\}$, AND DIFFERENT COMBINATIONS OF (h_1, h_2, K_1, K_2), WHEN DIFFERENT NUMBER OF PHASE-I SAMPLES m, EACH HAVING n = 5 OBSERVATIONS, ARE USED TO ESTIMATE THE IN-CONTROL PHASE-I PROCESS PARAMETERS

ATS_0 = 370						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
K_1	0.6352		0.6744		1.1315	
K_2	2.8650		2.8574		2.8602	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
500	354.93	54.85	353.90	55.38	354.43	55.36
800	360.89	42.65	359.92	43.06	360.45	43.04
900	362.07	40.04	361.11	40.43	361.63	40.41
950	**362.57**	38.90	361.61	39.27	362.14	39.25
1000	363.03	37.85	362.07	38.21	**362.60**	38.19
1050	363.44	**36.88**	362.49	**37.23**	363.02	**37.21**
1100	363.83	35.97	**362.88**	36.31	363.41	36.30
1150	364.18	35.13	363.23	35.46	363.76	35.45
1200	364.50	34.34	363.56	34.67	364.09	34.65
1250	364.80	33.60	363.86	33.92	364.39	33.91
+∞	370.00	0	370.00	0	370.00	0
ATS_0 = 500						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
K_1	0.6605		0.6682		1.1437	
K_2	2.9633		2.9634		2.9623	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
500	477.76	79.15	477.41	80.47	477.34	80.08
800	486.31	61.59	486.09	62.62	485.99	62.31
900	488.00	57.83	487.81	58.80	487.70	58.51
950	488.73	56.19	488.55	57.12	488.43	56.84
1000	489.39	54.67	489.22	55.58	489.10	55.31
1050	489.99	53.26	489.83	54.15	489.71	53.89
1100	**490.54**	51.96	**490.39**	52.82	**490.39**	52.82
1150	491.04	**50.74**	490.90	51.59	490.77	51.33
1200	491.51	49.61	491.37	**50.43**	491.25	**50.18**
1250	491.94	48.54	491.81	49.35	491.68	49.11
+∞	500.00	0	500.00	0	500.00	0

TABLE III: $AATS_0$ AND $SDATS_0$ VALUES FOR λ = 0.5, $ATS_0 \in \{370, 500\}$, AND DIFFERENT COMBINATIONS OF (h_1, h_2, K_1, K_2), WHEN DIFFERENT NUMBER OF PHASE-I SAMPLES m, EACH HAVING n = 5 OBSERVATIONS, ARE USED TO ESTIMATE THE IN-CONTROL PHASE-I PROCESS PARAMETERS

ATS_0 = 370						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
K_1	0.6272		0.6616		1.1555	
K_2	2.9852		2.9792		2.9783	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
200	361.18	95.79	360.73	96.94	360.61	96.48
250	**363.12**	85.22	**362.68**	86.23	**362.56**	85.83
300	364.51	77.50	364.07	78.42	363.96	78.06
350	365.56	71.54	365.12	72.39	365.01	72.06
400	366.37	66.77	365.93	67.56	365.83	67.25
450	367.03	62.84	366.59	63.58	366.49	63.29
500	367.56	59.52	367.12	60.22	367.03	59.95
800	369.45	46.80	369.02	47.35	368.93	47.14
1000	370.11	41.77	369.68	42.26	369.59	42.08
1100	370.35	39.80	369.93	40.27	369.84	40.09
1200	370.56	38.08	370.14	38.53	370.04	38.36
1300	370.74	**36.57**	370.31	**36.99**	370.22	**36.83**
1400	370.89	35.22	370.46	35.63	370.37	35.48
1500	371.02	34.01	370.60	34.41	370.50	34.26
1600	371.13	32.92	370.71	33.30	370.62	33.16
+∞	370.00	0	370.00	0	370.00	0
ATS_0 = 500						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
K_1	0.6468		0.6628		1.1542	
K_2	3.0744		3.0755		3.0712	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
200	487.75	137.05	487.94	139.46	487.46	138.42
250	**490.32**	121.86	**490.51**	123.98	**490.04**	123.07
300	492.18	110.78	492.37	112.70	491.91	111.88
350	493.58	102.25	493.78	104.01	493.32	103.26
400	494.68	95.41	494.88	97.05	494.43	96.35
450	495.57	89.78	495.76	91.31	495.31	90.66
500	496.29	85.03	496.49	86.48	496.04	85.87
800	498.86	66.83	499.08	67.97	498.63	67.49
1000	499.77	59.65	499.99	60.66	499.54	60.24
1100	500.11	56.83	500.32	57.79	499.88	57.39
1200	500.40	54.37	500.61	55.29	500.17	54.91
1300	500.63	52.21	500.85	53.09	500.41	52.72
1400	500.84	**50.28**	501.06	51.13	500.62	**50.78**
1500	501.02	48.56	501.24	**49.38**	500.80	49.03
1600	501.18	47.00	501.40	47.79	500.96	47.46
+∞	500.00	0	500.00	0	500.00	0

As noticed in Table I to Table III, the values of $SDATS_0$ diminish when the number of m increases. In order to achieve a stable AATS performance when the process parameters are estimated, SDATS is suggested to be around 10% of the ATS values. It is obvious from Table I to Table III that as λ decreases, the $SDATS_0$ values decreases for the same m. A smaller SDATS value indicates a lower level of practitioner-to-practitioner variability. By referring to Table I to Table III, the required m generally increases with an increase of ATS_0. For instance, when $\lambda = 0.2$, $(h_1, h_2) = (1.7, 0.3)$, and $n = 5$, we observe that $m = 1050$ are required when $ATS_0 = 370$, but it increases to $m = 1200$ when $ATS_0 = 500$ (see Table II). This is because the larger the ATS_0, the larger the values of K_1 and K_2, leading to wider warning and control limits.

From Table I to Table III, it is clear that depending solely on the AATS criterion will lead to select incorrect m, especially when λ is large. Using the SDATS criterion, a more stable practitioner-to-practitioner variability in control chart's performance will achieve. For example, when $\lambda = 0.5$, $ATS_0 = 370$, $(h_1, h_2) = (1.3, 0.1)$, and $n = 5$, we need $m = 250$ if AATS criterion is used and the corresponding $SDATS_0$ is 85.83, which is around 23% of the desired ATS_0 (see Table III). This 23% is very high and unfavorable. On the other hand, for the same case, we require $m = 1300$ if SDATS criterion is used and the corresponding $AATS_0$ is 370.22 (see Table III). This method of selection of m does not cause much issue as the $AATS_0$ (= 370.22) is very near to the desired ATS_0 (= 370). For $\lambda = 0.1$, $ATS_0 = 370$, $(h_1, h_2) = (1.3, 0.1)$, and $n = 5$, we require $m = 1000$ if SDATS criterion is used and the corresponding $AATS_0$ is 355.95 (see Table I). This $AATS_0$ value is about 96% of the desired ATS_0, which is still within 4% errors of the ATS_0 value.

B. *Out-of-Control Performances*

Table IV and Table V present the $AATS_1$ and $SDATS_1$ values, for $\lambda = 0.2$, various m, each with $n = 5$ observations, $(h_1, h_2) \in \{(1.5, 0.5), (1.7, 0.3), (1.3, 0.1)\}$ and $ATS_0 \in \{370, 500\}$, when $\delta \in \{0.2, 0.4\}$, respectively. For illustration, when $ATS_0 = 370$, $\delta = 0.2$, and $(h_1, h_2) = (1.5, 0.5)$, the combination of chart's parameters $(\lambda, K_1, K_2) = (0.2, 0.6352, 2.8650)$ as shown in Table II, is used to compute the $AATS_1$ and $SDATS_1$ values for various m in Table IV. The boldfaced entries represent the minimum number of m required to obtain the $AATS_1$ value within 2% errors of the corresponding ATS_1 value (when $m = +\infty$); while the boldfaced $SDATS_1$ value is within 10% of the corresponding ATS_1 value (when $m = +\infty$).

TABLE IV: $AATS_1$ AND $SDATS_1$ VALUES FOR $\lambda = 0.2$, $\delta = 0.2$, $ATS_0 \in \{370, 500\}$, AND DIFFERENT COMBINATIONS OF (h_1, h_2), WHEN DIFFERENT NUMBER OF PHASE-I SAMPLES m, EACH HAVING $n = 5$ OBSERVATIONS, ARE USED TO ESTIMATE THE IN-CONTROL PHASE-I PROCESS PARAMETERS

$ATS_0 = 370$						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
500	36.61	9.01	33.43	8.74	33.32	8.88
800	36.11	6.91	32.93	6.69	32.81	6.80
1000	35.94	6.12	32.76	5.92	32.64	6.02
1200	**35.83**	5.55	**32.65**	5.37	**32.53**	5.46
1400	35.76	5.11	32.58	4.95	32.46	5.03
1600	35.70	4.76	32.52	4.61	32.40	4.69
1800	35.65	4.48	32.47	4.33	32.35	4.40
2000	35.62	4.24	32.44	4.10	32.32	4.17
2200	35.59	4.04	32.41	3.90	32.29	3.97
2400	35.56	3.86	32.38	3.73	32.26	3.79
2600	35.54	3.70	32.36	3.58	32.24	3.64
2800	35.53	**3.56**	32.35	3.45	32.22	3.50
3000	35.51	3.44	32.33	3.33	32.21	3.38
3200	35.50	3.33	32.32	**3.22**	32.19	3.27
3400	35.49	3.23	32.31	3.12	32.18	**3.17**
3600	35.47	3.13	32.30	3.03	32.17	3.08
3800	35.47	3.05	32.29	2.95	32.16	2.99
4000	35.46	2.97	32.28	2.87	32.15	2.92
$+\infty$	35.19	0	32.02	0	31.90	0
$ATS_0 = 500$						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
500	42.85	11.19	38.61	10.76	38.58	10.96
800	42.20	8.56	37.96	8.21	37.92	8.37
1000	41.99	7.57	37.75	7.26	37.71	7.40
1200	**41.85**	6.86	37.61	6.58	37.57	6.70
1400	41.75	6.32	**37.51**	6.05	**37.47**	6.17
1600	41.67	5.89	37.43	5.64	37.39	5.75
1800	41.61	5.53	37.37	5.30	37.33	5.40
2000	41.57	5.24	37.33	5.01	37.29	5.11
2200	41.53	4.98	37.29	4.77	37.25	4.86
2400	41.50	4.76	37.26	4.56	37.22	4.65
2600	41.47	4.57	37.23	4.37	37.19	4.46
2800	41.45	4.40	37.21	4.21	37.17	4.29
3000	41.43	4.25	37.19	4.06	37.15	4.14
3200	41.41	**4.11**	37.17	3.93	37.13	4.01
3400	41.40	3.98	37.16	3.81	37.11	3.88
3600	41.38	3.87	37.14	**3.70**	37.10	3.77
3800	41.37	3.76	37.13	3.60	37.09	**3.67**
4000	41.36	3.66	37.12	3.51	37.08	3.57
$+\infty$	41.02	0	36.80	0	36.75	0

TABLE V: $AATS_1$ AND $SDATS_1$ VALUES FOR $\lambda = 0.2$, $\delta = 0.4$, $ATS_0 \in \{370, 500\}$, AND DIFFERENT COMBINATIONS OF (h_1, h_2), WHEN DIFFERENT NUMBER OF PHASE-I SAMPLES m, EACH HAVING $n = 5$ OBSERVATIONS, ARE USED TO ESTIMATE THE IN-CONTROL PHASE-I PROCESS PARAMETERS

$ATS_0 = 370$						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
200	7.48	1.40	6.09	1.20	5.60	1.17
250	**7.44**	1.23	6.06	1.05	5.57	1.03
300	7.42	1.11	**6.04**	0.95	5.54	0.93
350	7.40	1.02	6.02	0.87	**5.53**	0.85
400	7.39	0.95	6.01	0.81	5.51	0.79
450	7.38	0.89	6.00	0.76	5.51	0.74
500	7.37	0.84	5.99	0.72	5.50	0.70
550	7.36	0.80	5.99	0.68	5.49	0.67
600	7.36	0.76	5.98	0.65	5.49	0.64
650	7.35	**0.73**	5.98	0.63	5.48	0.61
700	7.35	0.71	5.97	**0.60**	5.48	0.59
750	7.35	0.68	5.97	0.58	5.48	0.57
800	7.34	0.66	5.97	0.56	5.47	**0.55**
850	7.34	0.64	5.96	0.54	5.47	0.53
900	7.34	0.62	5.96	0.53	5.47	0.51
$+\infty$	7.29	0	5.92	0	5.42	0
$ATS_0 = 500$						
(h_1, h_2)	(1.5, 0.5)		(1.7, 0.3)		(1.3, 0.1)	
m	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$	$AATS_0$	$SDATS_0$
200	8.11	1.57	6.45	1.32	5.88	1.28
250	8.06	1.38	6.41	1.16	5.84	1.12
300	**8.03**	1.25	**6.39**	1.04	5.82	1.01
350	8.01	1.15	6.37	0.96	**5.80**	0.93
400	8.00	1.07	6.36	0.89	5.78	0.86
450	7.98	1.00	6.35	0.83	5.77	0.81
500	7.97	0.95	6.34	0.79	5.76	0.76
550	7.97	0.90	6.33	0.75	5.76	0.73
600	7.96	0.86	6.33	0.71	5.75	0.69
650	7.95	0.82	6.32	0.68	5.75	0.66
700	7.95	**0.79**	6.32	0.66	5.74	0.64
750	7.95	0.76	6.31	**0.64**	5.74	0.62
800	7.94	0.74	6.31	0.61	5.74	0.60
850	7.94	0.72	6.31	0.60	5.73	**0.58**
900	7.94	0.69	6.31	0.58	5.73	0.56
$+\infty$	7.88	0	6.26	0	5.68	0

From Table IV, for (h_1, h_2) = (1.7, 0.3), it is observed that at least m = 1200 are needed to achieve an $AATS_1$ within 2% errors of the ATS_1 (m=+∞) when ATS_0 = 370; whereas at least m = 1400 are needed when ATS_0 = 500. On the other hand, the $SDATS_1$ results show that at least m = 3200 are needed in order to obtain an $SDATS_1$ value that is within 10% of the corresponding ATS_1 value (m=+∞) when ATS_0 = 370; while at least m = 3600 are required when ATS_0 = 500. The selection criterion based on the SDATS metric shows that a large amount of m is required to sufficiently reduce the variability in VSI EWMA $\bar{X}$ chart's performance. These results are consistent with the findings shown for other control charts, see [17], [21], [22], and [27].

Table VI shows the minimum number m, each of size n=5 required for the VSI EWMA $\bar{X}$ chart with estimated process parameters. The first row of each cell presents the minimum m for obtaining $AATS_1$ value within 2% errors of the corresponding ATS_1 value. The second row of each cell presents the minimum m for getting the $SDATS_1$ value within 10% of the corresponding ATS_1 value. Also, Table VI considers $\delta \in \{0.6, 0.8, 1.0\}$, $ATS_0 \in \{370, 500\}$, and $\lambda = 0.2$. Note that similarly as in Tables IV and V, the boldfaced $AATS_1$ in the first row of each cell is the $AATS_1$ value within 2% errors of the corresponding ATS_1 value (when m=+∞); while the boldfaced $SDATS_1$ in the second row of each cell is the $SDATS_1$ value within 10% of the corresponding ATS_1 value (when m = +∞). Because VSI EWMA $\bar{X}$ chart is effective in detecting small and moderate sustained shifts, we only consider $\delta \le 1.0$ in this paper. From Tables IV to VI, we notice that the minimum number m needed to achieve the acceptable $SDATS_1$ is large for small shifts ($\delta \le 0.4$). Nevertheless, the minimum number m drops significantly for moderate and large process mean shifts ($\delta \ge 0.6$).

TABLE VI: ($AATS_1$, $SDATS_1$) VALUES AND THE CORRESPONDING MINIMUM NUMBER OF PHASE-I SAMPLES m, EACH HAVING n = 5 OBSERVATIONS, WHEN $ATS_0 \in \{370, 500\}$, $\lambda = 0.2$, AND $\delta \in \{0.6, 0.8, 1.0\}$

	ATS_0 = 370		
(h_1, h_2)	(1.5, 0.5)	(1.7, 0.3)	(1.3, 0.1)
δ	(m, $AATS_1$, $SDATS_1$)	(m, $AATS_1$, $SDATS_1$)	(m, $AATS_1$, $SDATS_1$)
0.6	(100, **3.29**, 0.53)	(100, **2.55**, 0.43)	(110, **2.25**, 0.37)
	(260, 3.26, **0.32**)	(280, 2.52, **0.25**)	(290, 2.22, **0.22**)
0.8	(50, **1.90**, 0.34)	(60, **1.40**, 0.24)	(60, **1.22**, 0.22)
	(160, 1.88, **0.18**)	(180, 1.39, **0.14**)	(200, 1.21, **0.12**)
1.0	(40, **1.23**, 0.21)	(40, **0.86**, 0.16)	(40, **0.72**, 0.16)
	(120, 1.22, **0.12**)	(140, 0.85, **0.08**)	(190, 0.71, **0.07**)
	ATS_0 = 500		
(h_1, h_2)	(1.5, 0.5)	(1.7, 0.3)	(1.3, 0.1)
δ	(m, $AATS_1$, $SDATS_1$)	(m, $AATS_1$, $SDATS_1$)	(m, $AATS_1$, $SDATS_1$)
0.6	(100, **3.49**, 0.57)	(110, **2.65**, 0.43)	(110, **2.31**, 0.38)
	(270, 3.46, **0.34**)	(290, 2.62, **0.26**)	(290, 2.28, **0.23**)
0.8	(50, **2.01**, 0.36)	(60, **1.46**, 0.25)	(60, **1.26**, 0.23)
	(160, 1.98, **0.19**)	(180, 1.44, **0.14**)	(190, 1.24, **0.12**)
1.0	(40, **1.30**, 0.22)	(40, **0.90**, 0.17)	(40, **0.75**, 0.16)
	(120, 1.29, **0.12**)	(140, 0.89, **0.09**)	(190, 0.74, **0.07**)

IV. CONCLUSION

Process parameters are usually unknown in practical situations and they are estimated from an in-control Phase-I dataset. Hence, the SDATS is a favorable metric used to account for practitioner-to-practitioner variation. In this paper, we investigate the in-control and out-of-control performances of the VSI EWMA $\bar{X}$ chart with estimated process parameters, in terms of the AATS and SDATS. Recommendations for the required minimum number of Phase-I samples m are provided, in order to achieve the desired performance based on the SDATS metric.

Our simulation results show that the VSI EWMA $\bar{X}$ chart requires large number of Phase-I samples to achieve consistent chart's performance among practitioners. In addition, we observe that the VSI EWMA $\bar{X}$ chart designed with large values of smoothing constant λ, has high variability in the ATS distribution (see Tables I to III). Hence, large amount of Phase-I samples m is needed for large smoothing constant λ. However, the recommended large m is impractical; thus, future research may adopt the bootstrap method to adjust the control limits of the VSI EWMA $\bar{X}$ chart with estimated process parameters by using practical numbers of m.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

W.L. Teoh, M.B.C. Khoo, S.Y. Teh, Z.L. Chong, W.H. Moy, and H.W. You contributed to the theoretical approaches, simulation programming coding, and analysis of results. W.H. Moy computed the results from the developed simulation programs. W.L. Teoh wrote the paper. M.B.C. Khoo, S.Y. Teh, Z.L. Chong, and H.W. You review, edit, and improve the write up of the paper. All authors had approved the final version.

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