Robust Position Control of SM-PMSM Based on a Sliding Mode Current Observer

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***Abstract*—This paper propose a reduced-order sliding mode controller based on a sliding mode observer applied to a Surface Mounted Permanent Magnet Synchronous Motor (SM-PMSM). The external disturbances are considered in the design of the controller in order to provide good results and accuracy of the system. Additionally, the corresponding observer is used to estimate the rotor position. Simulations and experiment results are shown to confirm the effectiveness of the proposed controller and observer.**

***Index Terms*—SM-PMSM, sliding mode control, sliding mode observer, sensorless control, sigmoid function**

I. Introduction

Permanent Magnet Synchronous Motors (PMSM) are widely used in applications which needs a high performance and efficiency motor drives. Being this an electric machine that works in alternating current, little by little they are displacing induction motors in industry; however, in many cases the cost of the devices to control these motors is much greater than its counterpart induction motors.

In addition, in order to have good accuracy in the control system, it is necessary to have shaft mounted sensors such as encoders and tachometers which further increases costs and reduce reliability of the system. That is why in recent years has become a trend the use of controllers that do not require sensors (sensorless drivers) based on the idea of design observers which can estimate speed and position indirectly through the stator terminals [1] in a precise way and thus displacing classic control techniques such as Field – Oriented control.

Based on this idea, a bunch of different types of observers were proposed [2]-[6], such as the observer designed by Yong Chen *et al*. based on sliding mode techniques using the Adaptive Synchronization Filter (ASF) and Quadrature Phase-Lock Loop (QPLL) [2]. Furthermore, En Lu *et al*. considered the idea of developing a Second-order Non-singular Terminal Sliding Mode Observer (SNTSMO) to observe the changes of load disturbance [3].

However, the switching function used in Sliding Mode Techniques introduce a chattering phenomenon which could ruin the control system. Thus, Hongryel Kim *et al*. proposed the use of a sigmoid function in a sliding mode observer in order to improve results and accuracy of the system [4].

On the other hand, the controller design [3], [7]-[11] must have to take into account that these motors are exposed to parameters variations due to working conditions and disturbances that greatly affect the control system. Therefore, En Lu *et al*. also developed an anti-disturbances speed controller based on sliding mode techniques which is used to reduce the disturbance from the load [3]. Additionally, Dias Milena *et al*. designed a robust controller based on second order sliding mode techniques for parameters uncertainties and load torque perturbations [7] which give them good results of stability and convergence.

Finally, the focus of this article is to design a controller and observer law which provide us robustness and appropriate performances to control a PMSM. This paper is organized as follows: Mathematical Modelling of PMSM is explained in the following section. The subsequent section presents the design of the Sliding Mode Observer (SMO) and demonstration of its stability. Subsequently, according to previous section the Sliding Mode Controller (SMC) is designed. The simulation results of the SMO and the SMC are presented next. The final section presents conclusions of the study.

II. Mathematical Modeling of PMSM

Taking into account the coordinate transformation theory of AC motor systems, the three-phase stator circuit of a PMSM can be transformed into the equivalent two-phase circuit static or rotating coordinate system.

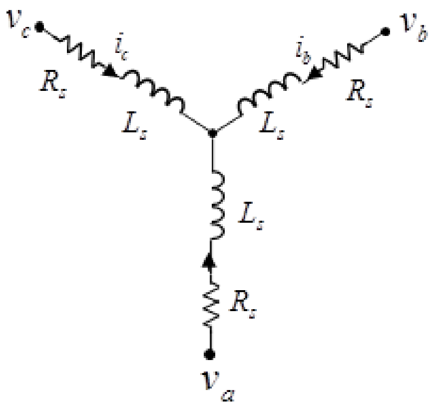

Fig. 1. Electric circuit of the PMSM stator [12]

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From the original three-phase coordinate system we have.

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \frac{d\psi_a}{dt} \\ \frac{d\psi_b}{dt} \\ \frac{d\psi_c}{dt} \end{bmatrix} \tag{1}$$

where Ψ_x are magnetic flux on each phase stator and v_x, i_x are voltage and current on the stator respectively as seen in Fig. 1.

In order to obtained the two phase stationary and rotating coordinate systems it is necessary to applied Clark and Park transformations respectively.

This transformations are given by the next matrices:

Clark Transformation:

$$\begin{bmatrix} \alpha \\ \beta \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{2}$$

Park Transformation:

$$\begin{bmatrix} d \\ q \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{3}$$

where θ is the electrical rotor angle.

Applying this transformations to equation (1) we obtained:

Two phase stationary system.

$$v_\alpha = R_s i_\alpha + \frac{d\phi_\alpha}{dt} \tag{4}$$

$$v_\beta = R_s i_\beta + \frac{d\phi_\beta}{dt} \tag{5}$$

$$\phi_\alpha = L_s i_\alpha + \phi_m \cos(\theta) \tag{6}$$

$$\phi_\beta = L_s i_\beta + \phi_m \sin(\theta) \tag{7}$$

where v_x, i_x and ϕ_x are voltage, current and fluxes in stationary reference frame, ϕ_m is the permanent magnet flux and R_s, L_s are the stator resistance and inductance.

Two phase rotating system.

$$v_d = L_s \frac{di_d}{dt} + R_s i_d - L_s N\omega i_q \tag{8}$$

$$v_q = L_s \frac{di_q}{dt} + R_s i_q + L_s N\omega i_d + N\omega\psi_m \tag{9}$$

where v_x, i_x are voltage and current in rotating reference frame, Ψ_m is the permanent magnet flux, ω and N are the mechanical rotor speed and number of pairs of poles respectively.

III. Design of the SMO

For this purpose, it is necessary to use Clark transformation because this transformation doesn't depend on the position of the rotor. Thus, using equations (4) to (7), the current space state model can be expressed as:

$$\dot{i}_\alpha = -\frac{R_s}{L_s} i_\alpha - \frac{1}{L_s} e_\alpha + \frac{1}{L_s} v_\alpha \tag{10}$$

$$\dot{i}_\beta = -\frac{R_s}{L_s} i_\beta - \frac{1}{L_s} e_\beta + \frac{1}{L_s} v_\beta \tag{11}$$

$$e_\alpha = -\phi_m \omega \sin(\theta) \tag{12}$$

$$e_\beta = \phi_m \omega \cos(\theta) \tag{13}$$

where e_x are the back emf in the stationary reference frame.

From equation (10) and (11), a sliding surface is selected as:

$$S(x) = \begin{bmatrix} S_\alpha(x) & S_\beta(x) \end{bmatrix}^T = \begin{bmatrix} \hat{\imath}_\alpha - i_\alpha \\ \hat{\imath}_\beta - i_\beta \end{bmatrix} \tag{14}$$

Thus, the current observer based on SMO has the form:

$$\dot{\hat{\imath}}_\alpha = -\frac{R_s}{L_s} \hat{\imath}_\alpha + \frac{1}{L_s} v_\alpha - \frac{1}{L_s} k sgn(\hat{\imath}_\alpha - i_\alpha) \tag{15}$$

$$\dot{\hat{\imath}}_\beta = -\frac{R_s}{L_s} \hat{\imath}_\beta + \frac{1}{L_s} v_\beta - \frac{1}{L_s} k sgn(\hat{\imath}_\beta - i_\beta) \tag{16}$$

where k is a constant gain observer and $sgn(x)$ is the sign function. However, in order to avoid chattering problems introduced by the sign function, a sigmoid function will be used:

$$H_{(s)} = \left[\frac{2}{(1+e^{-as})}\right] - 1 \tag{17}$$

Therefore, equations (15) and (16) will be express as.

$$\dot{\hat{\imath}}_\alpha = -\frac{R_s}{L_s} \hat{\imath}_\alpha + \frac{1}{L_s} v_\alpha - \frac{1}{L_s} kH(\hat{\imath}_\alpha - i_\alpha) \tag{18}$$

$$\dot{\hat{\imath}}_\beta = -\frac{R_s}{L_s} \hat{\imath}_\beta + \frac{1}{L_s} v_\beta - \frac{1}{L_s} kH(\hat{\imath}_\beta - i_\beta) \tag{19}$$

A. *Stability of the SMO*

In order to study the stability of the SMO designed, the next Lyapunov quadratic function is selected:

$$V = \frac{1}{2}(S_\alpha^2 + S_\beta^2) \tag{20}$$

Thus, the stability condition for the SMO is presented as:

$$\dot{V} = S_\alpha \dot{S}_\alpha + S_\beta \dot{S}_\beta \leq 0$$

$$\dot{V} = -\frac{R_s}{L_s}\left(S_\alpha^2 + S_\beta^2\right) + \frac{1}{L_s}\left(e_\alpha S_\alpha - S_\alpha kH(S_\alpha)\right) + \frac{1}{L_s}\left(e_\beta S_\beta - S_\beta kH\left(S_\beta\right)\right) \leq 0$$

Consequently, the observer gain should satisfy the next inequality condition:

$$k \geq \max(|e_\beta|, |e_\alpha|) \tag{21}$$

where k should be selected sufficiently large.

Therefore, when the sliding surface is achieved ($s_{(x)}=0$), the variables e_α and e_β will be given by the equivalent control method as:

$$e_\alpha = kH(\hat{\imath}_\alpha - i_\alpha) \tag{22}$$

$$e_\beta = kH(\hat{\imath}_\beta - i_\beta) \tag{23}$$

And the mechanical rotor position will be given by:

$$\theta_r = \frac{1}{N} arctan(-\frac{e_\alpha}{e_\beta}) \quad (24)$$

IV. DESIGN OF THE SMC

Using equation (8), (9) and mechanical rotor dynamics, the current space state model is presented as:

$$\dot{i}_d = -\frac{R_s}{L_s} i_d + N\omega i_q + \frac{1}{L_s} v_d \quad (25)$$

$$\dot{i}_q = -\frac{R_s}{L_s} i_q - N\omega i_d - \frac{Kt}{L_s}\omega + \frac{1}{L_s} v_q \quad (26)$$

$$\dot{\omega} = \frac{3}{2}\frac{Kt}{J_m} i_q - \frac{B_m}{J_m}\omega_m - \frac{T_L}{J_m} \quad (27)$$

where J_m, K_t, B_m and T_L are the rotor inertia, torque constant, friction coefficient and the load torque respectively.

In equation (27) it can be seen that i_d doesn't affect the position dynamics. Thus, in order to achieve a superior performance [3], a PI controller with the reference i_d=0 is implemented. Therefore, the close loop control system for the PMSM is develop as in Fig. 2.

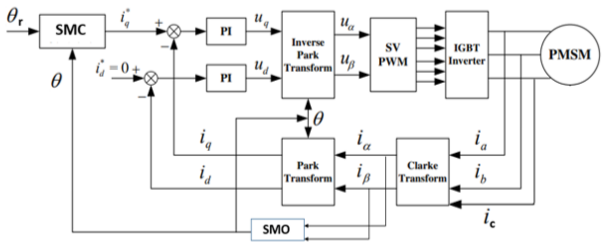

Fig. 2. Close loop control system of PMSM.

Thus, the SMC will be based on the error signal $e=\theta_r-\theta$ and the space state model is expressed as:

$$e = \theta_r - \theta = x_1 \quad (28)$$

$$\dot{x}_1 = \dot{\theta}_r - \dot{\theta} = x_2 \quad (29)$$

$$\dot{x}_2 = \ddot{\theta}_r - \ddot{\theta} = -\frac{3}{2}\frac{Kt}{J} i_q - \frac{B}{J} x_2 + d_{(t)} \quad (30)$$

$$d_{(t)} = \ddot{\theta}_r + \frac{B}{J}\dot{\theta}_r + \frac{T_L}{J_m} \quad (31)$$

A. Selection of the Sliding Mode Surface

From the space state error model given in equations (29) and (30), arranging in a matrix form:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{B}{J}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ -\frac{3}{2}\frac{Kt}{J}\end{bmatrix} i_q + \begin{bmatrix}0\\ 1\end{bmatrix} d_{(t)} \quad (32)$$

The space state model presented is in a regular form, so the sliding surface will be given by:

$$s(t) = [S_1 \;\; S_2]\begin{bmatrix}x_1\\ x_2\end{bmatrix} = S_2[M \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \quad (33)$$

Selecting S_2=1, M will be found by quadratic minimization method. Thus, having Q=I_{2x2}, are defined:

$$\breve{Q} = Q_{11} - Q_{12}Q_{22}^{-1}Q_{12}^T \quad (34)$$

$$\breve{A} = A_{11} - A_{12}Q_{22}^{-1}Q_{12}^T \quad (35)$$

where A_{11}=0 and A_{12}=1. Replacing (34) and (35) in Riccati equation (36), M will be given by:

$$P\breve{A}^T + \breve{A}P + \breve{Q} - PA_{12}Q_{22}^{-1}A_{12}^T P = 0 \quad (36)$$

$$M = Q_{22}^{-1}Q_{12}^T + Q_{22}^{-1}A_{12}^T P \quad (37)$$

As a result of substituting all Q and A element values in (37), the value of M is:

M=1

And finally:

$$s(t) = [1 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix} = x_1 + x_2 \quad (38)$$

B. Selection and Stability of de SMC

From reachability condition:

$$\dot{s} = -\eta sign(s) \quad (39)$$

Differentiating (38) and replacing (39), controller equation is given as follows:

$$i_q = \frac{2}{3}\frac{J}{Kt}[\left(1 - \frac{B}{J}\right)x_2 + \eta sign(s) + d_{(t)}] \quad (40)$$

In order to analyze the stability of the SMC, we have from the reachability condition in its other form the next equation:

$$s\dot{s} = s(x_2 - \frac{3}{2}\frac{Kt}{J} i_q - \frac{B}{J}x_2 + d_{(t)})$$

Replacing (40):

$$s\dot{s} = s(-\eta sign(s))$$
$$s\dot{s} < -|s|\eta$$

Therefore, for every η real positive the SMC is asymptotically stable.

V. SIMULATION RESULTS

In order to validate the effectiveness of our sliding mode controller and observer, the program Matlab/Simulink was used to implement the mathematical equations of the PMSM, SMO and SMC. The values used for simulations are presented in Table I.

TABLE I: SIMULATION PARAMETERS

Stator resistance Rs	0.4578 Ω
Stator inductance Ls	0.00333 mH
Permanent magnet flux	0.171 Wb
Rotor Inertia	0.001469 $Kg.m^2$
Friction Coefficient	0.0003035 $Kg.m^2$ /s
N °of poles	8
Slope of sigmoid function	2
SMO gain	100
SMC gain	1000

Firstly, the SMO was simulated with a PI controller as shown in Fig. 3 in order to prove its effectiveness. The results given by the observer with no load torque from the rotor are presented in Fig. 4 and Fig. 5. Additionally, with a load torque of 5 N.m results are presented in Fig. 6 and Fig. 7. The observer designed shows reliable estimations of the position; however, this reliability is reduced when a load torque is introduced in the system but it is still effectiveness.

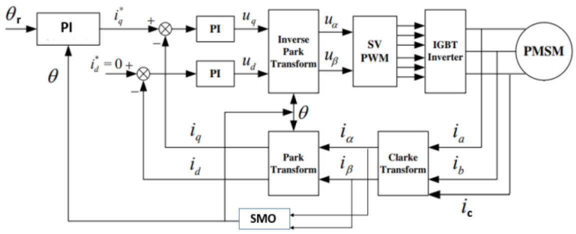

Fig. 3. PI controller with SMO.

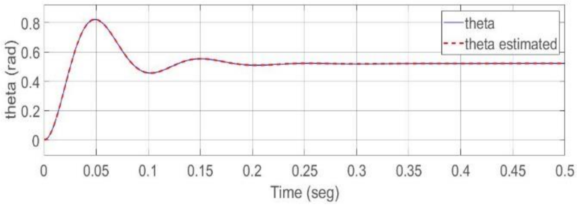

Fig. 4. Actual and estimated position with no load.

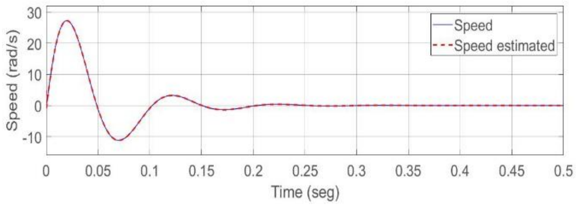

Fig. 5. Actual and estimated speed with no load.

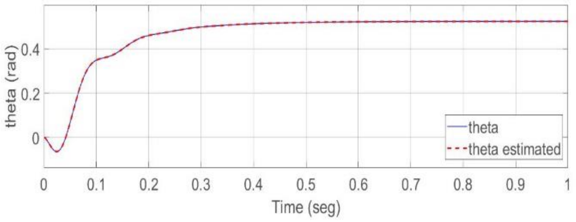

Fig. 6. Actual and estimated position with load.

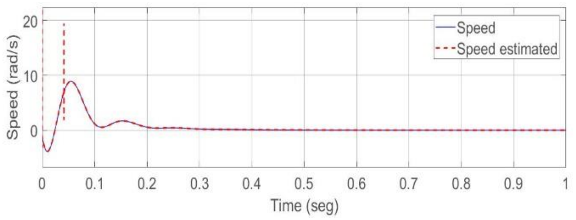

Fig. 7. Actual and estimated speed with load.

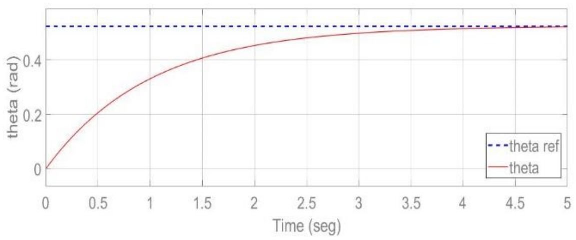

Fig. 8. Reference and actual theta with no load.

On the other hand, the SMC was proved without the observer in cases of position and tracking control with no load in Fig. 8, Fig. 9 and results with a load torque of 5 N.m in Fig. 10 and Fig. 11. The controller presents high quality results in both cases tracking and position control. Without any error in both cases, the controller designed shows a great robustness.

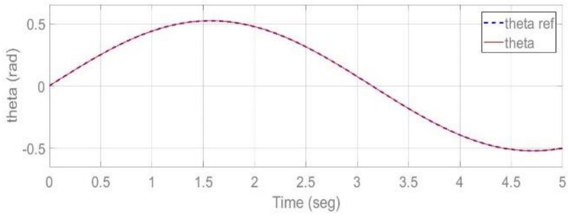

Fig. 9. $\theta_{ref}=\pi\sin(t)/6$ and no load.

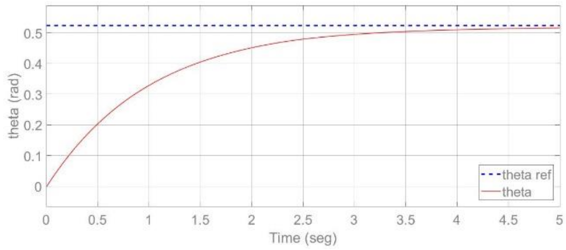

Fig. 10. Reference and actual theta with load.

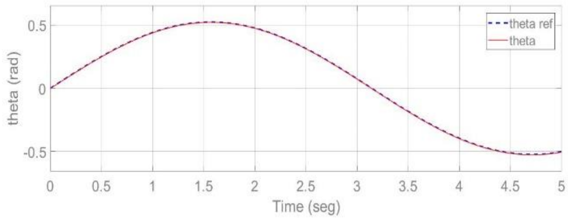

Fig. 11. $\theta_{ref}=\pi\sin(t)/6$ with load.

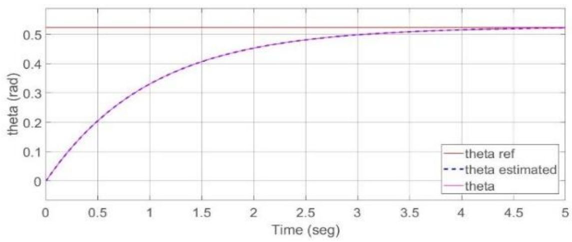

Fig. 12. Reference,actual and estimated theta with no load.

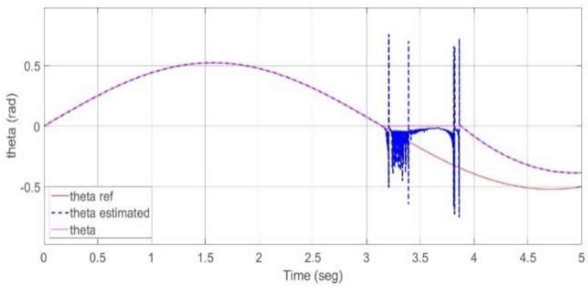

Fig. 13. $\theta_{ref}=\pi\sin(t)/6$ and no load.

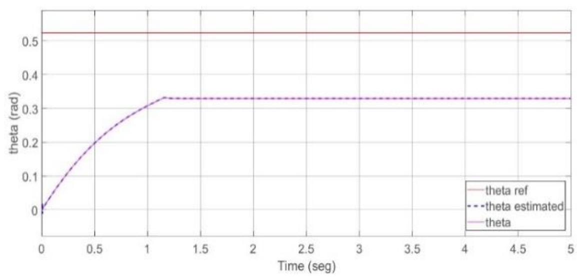

Fig. 14. Reference,actual and estimated theta with load.

Once the SMO and the SMC were verified individually, the complete system SMO with SMC was tested in order to validate the effectiveness of the system, obtaining the results with no load shown in Fig. 12, Fig. 13 and results with a load torque of 5 N.m in Fig. 14 and Fig. 15. The complete system shows good results in position and tracking control considering the case of no load; however, adding a load torque introduce a considerable error signal which cannot be fixed.

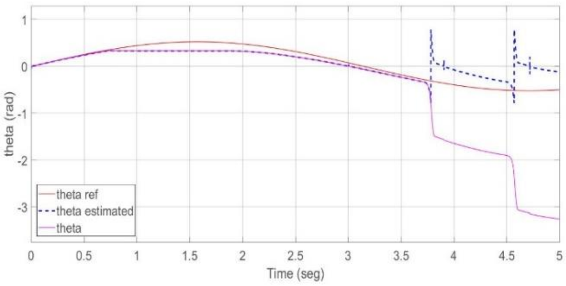

Fig. 15. $\theta_{ref}=\pi\sin(t)/6$ with load.

VI. Conclusions

The proposed SMO has presented a good effectiveness in estimating position. However, introducing this observer into a PI controller was not a good option because of the sign function; so in order to overcome this, a sigmoid function replaced the sign function in the observer.

The SMC considering load and other disturbances performs very well in cases with no load and load torque. Additionally, this controller was used as a tracking controller and it had high quality results. However, combining the SMC and SMO doesn't give same results as only SMC considering load. This last results show us that the observer needs to be improved in order to increase the robustness of the complete system.

Conflict of Interest

The author declares no conflict of interest.

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