Octagonal Photonic Crystal Fibre with Golden Ratio Principle as a Dispersion Compensating

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Abstract—An octagonal 5-layer air-hole dispersion compensating photonic crystal fibre is presented in this paper. The design is based on the golden ratio principle where the pitch to air hole diameter ratio is kept at 1.618. Numerical analysis using full-vector Finite Element Method (FEM) with a perfectly matched layer boundary condition is used to investigate optical properties of the Photonic Crystal Fibre (PCF). At its optimum pitch value, high negative dispersion of $-295 \text{ps/(nm.km)}$ and low confinement loss of $0.00755 \text{dB/km}$ is achieved at 1550 nm wavelength. Similarly, the proposed PCF also yields negative dispersion and low confinement loss on the E+S+C+L+U wavelength bands and partially on the O band with optimum pitch. These properties point to the suitability of the proposed PCF for potential application as a dispersion compensating fibre.

Index Terms—Chromatic dispersion, confinement loss, dispersion compensating fibre

I. INTRODUCTION

There is an ever-increasing demand for high-speed long haul transmission links, made possible using fibre optic technology. However, information carrying capacity of the fibre is still very much limited by pulse broadening due to chromatic dispersion [1]. To overcome this difficulty, dispersion compensation techniques become indispensable, with chirped in-fibre Bragg gratings [2], planar light wave circuit dispersion equalizer [3] and dispersion compensating fibres (DCFs) [4]. Among techniques that have been used. DCFs are generally preferred for dispersion compensation due to its reliability, simplicity and suitability for broadband dispersion compensation [4].

Since the first introduction of photonic crystal fibre (PCF), many advancements have been made to improve structure of such fibres to suit different applications [5]. Design flexibility of PCFs allows one to manipulate several parameters such as lattice type, lattice pitch, air hole diameter and shape as well as the refractive index of the material, to obtain unique and desirable dispersion properties that are impossible to achieve with conventional fibres [6]. High negative dispersion is one advantageous property of PCFs that makes it useful for mitigation of chromatic dispersion in long haul transmission links. This has led to surging developments of PCF-based DCFs within the past decade [1], focusing on designing PCFs with high negative dispersion property with minimal loss.

Many PCF designs can be found in the literature. Ani et al. [7] proposed a circular lattice PCF producing an average negative dispersion of $-124.0 \text{ps/(nm.km)}$ with confinement loss of less than $10^{-6} \text{dB/km}$. In [8], a double-hole assisted core encapsulated in a square lattice cladding has been proposed, with negative dispersion of $-150 \text{ps/(nm.km)}$ and loss of less than $10^{-5} \text{dB/km}$. High negative dispersions of $-506 \text{ps/(nm.km)}$ and $-558.96 \text{ps/(nm.km)}$ in the 1460 to 1675 nm wavelength range, have been demonstrated using a modified octagonal PCF [9] and a decagonal PCF structure [10], respectively. However, losses are a bit higher; in the order of $10^{-3} \text{dB/km}$ [9] and $10^{-2} \text{dB/km}$ [10].

Ahmed et al. [11] presented a square lattice PCF that exhibits an ultrahigh negative dispersion of $-1083 \text{ps/(nm.km)}$ at 1550 nm wavelength, although the confinement loss; important property for DCF, has not been investigated. Meanwhile, an octagonal structure PCF with negative dispersion coefficient of $-3339 \text{ps/(nm.km)}$ with confinement loss in the order of $10^{-2} \text{dB/km}$ has been reported by Roy et al. [12].

The above literatures [7]-[12] indicate that for DCF, the design is a trade-off between negative dispersion and confinement loss; whereby to obtain higher negative dispersion would inevitably lead to higher loss. Additionally, design complexity also needs to be taken into account. Although the designs in reference [7]-[12] exhibited high negative dispersion and low confinement loss, the designs contained multiple air hole sizes and complex geometrical structure that would make fabrication process very difficult. As such, it is important that not only the design DCF should have high negative dispersion and low confinement loss, the design must also be made simpler.

The principle of golden ratio may be used for designing such PCF. Considered to be pleasing to the human eye, golden ratio has been applied to many aspects of nature such as body proportions of living things, insects, the arrangement of leaves in the stem of a plant, and the model of the universe. Not only that, the golden ratio is responsible for the basis of many simple but yet strong structures and architectures around the world, namely the Pyramids of Giza and the Parthenon, all built...
many years ago and still standing strong today [13], [14]. The principle has not been applied in many PCFs generally, with the exceptions for references [15], albeit not specifically as dispersion compensating fibre. Because of this, the golden ratio is applied to the structure of the air holes in the proposed PCF-based DCF.

In this paper, a simple octagonal PCF structure, based on the golden ratio, with 5 air-hole rings based on silica material is presented. The fibre gives sufficiently high negative dispersion of $-295.2 \text{ ps/(nm.km)}$ and low confinement loss of less than $10^{-3} \text{ dB/km}$ at $1550 \text{ nm}$ wavelength. Effective area and non-linear coefficient of the fibre are also studied. Due to its design simplicity, it can be fabricated easily using current fabrication technique.

II. GEOMETRY OF THE PROPOSED DESIGN

Geometrical cross-section of the proposed 5-ring octagonal PCF is presented in Fig. 1, with $8n$ air holes on the $n^{th}$ ring of the fibre, where $n = 1$ to 5. Its geometry is defined by air hole diameter, $a$, and pitches; with $A$ and $A_t$ denoting pitches between adjacent air holes on the same ring and on different rings, respectively. Pitches $A$ and $A_t$ are related by $A = 0.765A_t$. In order to strengthen the fibre structure, the golden ratio principle is followed in the design of the fibre. Denoting the golden ratio [16] by the Greek letter $\phi$, with a mathematical value of 1.618, pitch to diameter ratio ($A/d$) of the fibre is fixed, such that:

\[ A/d = 1.618 \] (1)

![Fig. 1. Cross-sectional structure of the proposed PCF.](image)

Silica is used as background material for the proposed fibre. Refractive indexes of the dielectric material and air are 1.45 and 1, respectively. A perfectly matched layer (PML) surrounds the PCF structure [17], absorbing lights which may pass from the core to the cladding and hence, suppressing any unwanted boundary reflection coming from the electromagnetic wave. A fine mesh type is used, which produces a total of 18900 elements, composed of 3124 edge elements, 488 vertex elements and 37738 triangular elements. Average element quality of 0.7943 is obtained from the mesh.

III. ANALYSIS METHOD

Full vector finite element method (FEM) with PML boundary condition is used to simulate and analyse optical properties of the proposed PCF.

Effective refractive index, $n_{\text{eff}}$ is calculated by using the three-terms Sellmeier equation [17], [18], given by

\[ n_{\text{eff}} = \sqrt{1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3}} \] (2)

where $\lambda$ is wavelength, and $B_1$, $B_2$, $B_3$, $C_1$, $C_2$, and $C_3$ represent Sellmeier coefficients. For every wavelength, effective refractive index of silica varies according to (2).

Effective refractive index is then used to calculate other optical properties of the proposed PCF such as chromatic dispersion $D$, confinement loss, $L_c$, effective area $A_{\text{eff}}$ and nonlinearity coefficient $\gamma$ [19]:

\[ D(\lambda) = \frac{\lambda}{c} \frac{d^2 \text{Re}[n_{\text{eff}}]}{d\lambda^2} \] (3)

\[ L_c = 8.686 \times 10^{-12} |k_0 \text{Im}[n_{\text{eff}}]| \times 10^3 \] (4)

\[ A_{\text{eff}} = \frac{\iint |E|^2 dxdy}{\iint |E|^0 dxdy} \] (5)

\[ \gamma = \frac{2\pi n_2^2}{\lambda} \frac{\text{Re}[n_{\text{eff}}]}{A_{\text{eff}}} \times 10^3 \] (6)

where Re[$n_{\text{eff}}$] and Im[$n_{\text{eff}}$] are real and imaginary parts of effective refractive index, $n_{\text{eff}}$, $k_0 = 2\pi/\lambda$ is the free space wave number, $E$ is amplitude of the transverse electric field propagating inside the PCF and $n_2 = 3.0 \times 10^{-20} \text{ m}^2/\text{W}$ is the non-linear refractive index of pure silica material [20], [21].

Relative dispersion slope, RDS and total residual dispersion, $D_T$ are properties that are equally important to determine suitability of the proposed PCF to function as dispersion compensating fibre. These properties are calculated using [20], [22]:

\[ \text{RDS} = \frac{S_{\text{SMF}}}{D_{\text{SMF}}} - \frac{S_{\text{DCF}}}{D_{\text{DCF}}} \] (7)

\[ D_T = D_{\text{SMF}}L_{\text{SMF}} + D_{\text{DCF}}L_{\text{DCF}} \] (8)

where $D_{\text{SMF}}$ and $D_{\text{DCF}}$ are dispersion coefficients of single mode fibre and dispersion compensating fibre, respectively. $D_T$ is derived for particular wavelength $\lambda$ using (3). $L_{\text{SMF}}$ and $L_{\text{DCF}}$ are lengths of single mode fibre and dispersion compensating fibre, respectively. $L_{\text{SMF}}$ is
taken to be 40 km, standard length for analysis of
dispersion compensating fibre whilst $L_{DCF}$ is chosen such
that total residual dispersion is zero at 1.55 $\mu$m
wavelength.

$S_{SMP}$ and $S_{DCF}$ are the dispersion slopes of single mode
fibre and dispersion compensating fibre, respectively, both of which can be calculated using [20], [22]:

$$S_{SMP}, \text{ or } S_{DCF} = \frac{dD(\lambda)}{d(\lambda)}$$  \(9\)

IV. SIMULATION RESULTS

A. Effective Refractive Index

Simulations are performed for the proposed PCF for
four different values of pitch $\Lambda = 0.9 \mu$m, $\Lambda = 1.0 \mu$m,
$\Lambda = 1.06 \mu$m and $\Lambda = 1.1 \mu$m, in order to investigate
different optical properties of the proposed PCF.
Effective refractive index of the proposed PCF is
obtained for different values of $\Lambda$ and wavelength, used as basis for analysis, given in Fig. 2. It can be seen that
effective refractive index $n_{eff}$ decreases as wavelength
increases, with low $\Lambda$ giving lower effective refractive
index. When pitch value is larger in that time amount of
silica in the cladding region is higher that bring core and
cladding effective refractive index difference larger.
Therefore, effective refractive index value is higher in
higher pitch value. On the other hand, core and cladding
effective refractive index will be reduced in case of small
pitch value because less amount of silica will be existed
in the cladding region that bring core and cladding
effective refractive index difference smaller. Thereby in
smaller pitch value effective refractive index value is lower.

B. Chromatic Dispersion

Effective refractive index $n_{eff}$ is then used to determine
other optical properties. Chromatic dispersion is an
unavoidable phenomenon found in fibre optic
communication, whereby light with a range of wave-
length passing through the fibre travels at slightly
different speeds, causing them to arrive at their destina-
tions at slightly different times. This results in broadening
of pulses, especially in long haul transmissions, where
the information capacity of the fibre can be significantly
reduced. Relationship between chromatic dispersion of
the proposed PCF and wavelength, for different values of
$\Lambda$ is given in Fig. 3.

It can be seen that chromatic dispersion decreases with
increasing wavelength, with lower $\Lambda$ producing higher
negative dispersion. Negative dispersion values of $-420.3$
ps/(nm-km), $-295.2$ ps/(nm-km) and $-224.6$ ps/(nm-km)
are obtained with $\Lambda$ at 1.0 $\mu$m, 1.06 $\mu$m and 1.1 $\mu$m,
respectively, at 1.55 $\mu$m wavelength. Similarly, it can be
seen that the proposed PCF exhibits the highest negative
dispersion at $-655.1$ ps/(nm-km) when $\Lambda = 0.9 \mu$m at 1.55
$\mu$m wavelength.

C. Confinement Loss

Fig. 4 gives the confinement loss of the proposed PCF
for different wavelengths and $\Lambda$. The confinement loss
represents the light loss due to the mode propagating out-
side the centre core of the fibre. Generally, the confinement
loss increases with wavelength and $\Lambda$, as seen from the
figure. Although $\Lambda = 0.9 \mu$m gives the highest negative
dispersion, the confinement loss is also particularly high
for that value of $\Lambda$. Increasing $\Lambda$ allows a reduction in the
confinement loss. At 1.55 $\mu$m wavelength, the confinement
loss for $\Lambda$ at 0.9 $\mu$m is 0.872 dB/km, in contrast to
losses of 0.04588 dB/km, 0.00755 dB/km and 0.00227
dB/km for $\Lambda$ at 1.0 $\mu$m, 1.06 $\mu$m and 1.1 $\mu$m, respectively;
which are in the order of less than $10^{-2}$ dB/km.

D. Effective Area and Nonlinear Coefficient

Effective area is used to measure non-linear properties
of the fibre; with low effective area providing high
density of power which in turn signifies nonlinear effect
[23]. PCFs with high nonlinear coefficient are needed for
soliton application, supercontinuum generation, optical
code division multiple access and other nonlinear
applications [12]. Additionally, PCFs with higher
nonlinear coefficients are found to be more suitable for
dispersion compensation [12].
Fig. 5 and Fig. 6 give the relationship between wavelength and effective area, and wavelength and nonlinear coefficient, respectively, for different values of $A$. Generally, effective area increases with wavelength and $A$. Conversely, nonlinear coefficient decreases with wavelength and $A$, with steeper relationship between nonlinear coefficient and wavelength, at low $A$. Lowest effective area, 1.43 $\mu m^2$, is obtained with $A=0.9$ $\mu m$ at the 1.55 $\mu m$ wavelength. Due to the inverse relationship between effective area and nonlinear coefficient, $A=0.9$ $\mu m$ gives the highest nonlinear coefficient of 85.1 $W^{-1}km^{-1}$.

E. Relative Dispersion Slope

Dispersion slope $S_{DCF}$ of the dispersion compensating fibre is essential in calculating the slope mismatch with single mode fibres (SMF). Taking relative dispersion slope, RDS of a SMF to be 0.0036 $nm^{-1}$, RDS of the DCF needs to be close to the assumed RDS value of the SMF at 1.55 $\mu m$ wavelength, in order for the DCF to be applicable. RDS of the DCF at different values of $A$ are tabulated in Table I. It can be seen that $A=1.06$ $\mu m$ gives RDS of 0.0036 $nm^{-1}$, exact RDS of SMF, and as such $A=1.06$ $\mu m$ is used henceforth as the optimum $A$ parameter. Fig. 7 shows the relationship between RDS of the proposed fibre and wavelength at $A=1.06$ $\mu m$; with RDS decreasing with increase in wavelength.

<table>
<thead>
<tr>
<th>Pitch, $A$ ($\mu m$)</th>
<th>RDS (nm$^{-1}$)</th>
</tr>
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<tbody>
<tr>
<td>0.9</td>
<td>0.0013</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0027</td>
</tr>
<tr>
<td>1.06</td>
<td>0.0036</td>
</tr>
<tr>
<td>1.1</td>
<td>0.0044</td>
</tr>
</tbody>
</table>

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$F$. Residual Dispersion

Negative dispersion property of dispersion compensating fibre is needed to compensate for the positive dispersion of single mode fibre, over wide range of wavelength. However, even after compensation, there still may be some left over dispersion, referred to as total residual dispersion, $D_r$, which needs to be kept as low as possible. Fig. 8 depicts $D_t$ for different wavelengths at $A=1.06$ $\mu m$; with residual dispersion equal to zero at 1.55 $\mu m$ wavelength and length of dispersion compensating fibre $L_{DCF}=2.24$ $km$ for a single mode fibre $L_{SMF}=40$ $km$.

V. Conclusion

A single mode octagonal PCF with 5 air-hole rings has been proposed; following golden ratio principle to strengthen its structure whilst still keeping the design simple. Full vectorial finite element method has been used to numerically analyse different optical properties of the proposed PCF, for different values of $A$. With $A=0.9$ $\mu m$, the proposed PCF exhibits the highest negative dispersion and nonlinear coefficient of $-655.1$ ps/(nm.km) and 85.1 $W^{-1}km^{-1}$, respectively, with the lowest effective area of 1.3 $\mu m^2$. However, minimum confinement loss of 0.00227 $dB/km$ is obtained with $A=1.1$ $\mu m$. As such, the study has shown that it is not possible to simultaneously obtain high negative dispersion with low confinement loss due to trade-off between these two properties.

It has been shown that RDS of the proposed PCF is identical to that of single mode fibre, at $A=1.06$ $\mu m$ and as such, the value has been adopted as the recommended pitch of the proposed PCF. With this pitch value, sufficiently high negative chromatic dispersion of $-295.2$ ps/(nm.km) and low confinement loss of 0.00755 $dB/km$ are obtainable, at 1.55 $\mu m$ wavelength. Effective area of
1.88 μm² and non-linear coefficient of 64.66 W⁻¹km⁻¹ may also be obtained. Additionally, the proposed PCF is able to cover a wide range of bandwidth, notably the E+S+C+L+U, and the O band partially. This makes the fibre suitable for not only dispersion compensation, but also for non-linear applications. Moreover, design simplicity of the proposed PCF makes its fabrication feasible.

CONFLICT OF INTEREST
The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS
Fatin Nadhirah and Izaddeen Yakasai designed the model, carried out the simulation, analysed the data and performed the numerical calculations. Fatin Nadhirah and Izaddeen Yakasai wrote the manuscript with the help of Pg Emeorylarision Abas, Saifullah Abu Bakar, and Feroza Begum. Pg Emeorylarision Abas and Feroza Begum provided critical feedback and helped shape the research, analysis and manuscript. Feroza Begum conceived the original idea, contributed to the interpretation of the results and supervised the project. All authors discussed the results and contributed to the final version of the manuscript.

REFERENCES
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Feroza Begum received her Ph.D. degree in electronics and information engineering and M.E. degree in electronic material engineering from University of the Ryukyus, Japan in 2007 and 2004, respectively. She received her B.E. degree in electrical and electronics engineering from Dhaka University of Engineering and Technology, Bangladesh in 1998. Currently she is working as an assistant professor in system engineering at Faculty of Integrated Technologies, Universiti Brunei Darussalam (UBD) in Brunei Darussalam. Before joining to UBD, she worked in Dalian Polytechnic University, China and numerous universities in Bangladesh as a Faculty Member. She received two postdoctoral research fellowship: one is from Japan Society for the Promotion of Science for the duration of August 2009 to September 2011 from and another one is Marubun Research Promotion Foundation, Japan from 2008 to 2009. She published more than 110 peer review papers including journals, conferences and book chapter and holds one patent. She is engaged in research on optical fibre and photonics, photonic crystal fibres, optical fibre communications, optical coherence tomography, terahertz wave technology and optical sensors. Dr. Begum received Excellent Thesis Advisor Award at Dalian Polytechnic University in China and Best Presenter Award in different conferences. Begum served as a Technical Committee Member and International Scientific Committee Member in different conferences. Begum is reviewer of Optics and Photonics related journals.