BER Performance of Cooperative MIMO Systems with Half-Duplex Decode and Forward Relaying

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Abstract—The joint combination of Multiple-Input Multiple-Output (MIMO) technology and relay, known as cooperative MIMO systems, can extremely boost the spectral efficiency of wireless systems. Relevant researches regarding protocol, capacity limit and power allocation for such a system can be easily found in the literature. However, a study on Bit Error Rate (BER) performance which is crucial for this system has received less attention. This motivates us to perform further investigation on this topic. In order to conduct the simulation, the system with single relay and half-duplex decode-and-forward protocol, which is quite simple from practical point of view, is considered in this paper. Our results indicate that it is worth to have relay in MIMO system since relay can provide power saving at least 8 dB comparing with MIMO system without relay at BER of $10^{-6}$. Moreover, the use of simple relaying strategy called silent source in multiple-access (MAC) mode does not degrade the BER performance in MIMO relay systems.

Index Terms—cooperative communication, decode-and-forward, half-duplex, MIMO, MIMO relay

I. INTRODUCTION

At present, wireless technology is the most popular form of communication due to its ease of use and it has been developed rapidly in response to demand of users. The main technologies used to increase performance of wireless communications are multiple input multiple output technology [1] and cooperative communications [2]. Both technologies currently attract a large amount of attention from researchers.

For Multiple-Input Multiple-Output (MIMO) systems, multiple antennas are equipped at both transmitter and receiver. MIMO systems can improve the wireless communication performance by exploiting multipath [3]. Under the same bandwidth and transmission power utilization, MIMO systems have better data rate and Bit Error Rate (BER) comparing with conventional systems, i.e., systems with single antenna [4].

In 1971, the idea of cooperative communication was first presented by Van Der Meulen [5], the foundation of relay channel has been established in this research. This work stated that both the transmitter and receiver nodes can be aided by a relay node to enhance the system performance. The relay nodes must be located between the source and the destination. Fig. 1 shows an example of cooperative communication which has three intermediate relay nodes. With the help of relay nodes, the channel capacity is increased [6]. Furthermore, the BER of the overall system is also improved [7], [8].

Due to the advantages of MIMO systems and relay nodes mentioned above, researchers have the idea to jointly combine both technologies to further increase the systems performance and this communication system is known as MIMO relay systems [9]. Over recent years, MIMO relays system is recognized as a promising candidate technology for wireless communication standards, such as Long Term Evolution (LTE) or 5G wireless communication systems [10], [11].

Over the years, many aspects of MIMO relay systems have been investigated. There have been many computational techniques to search for the best power allocation in MIMO relay systems [12], [13]. The channel capacity of the MIMO relay was first analyzed in 2005 by Wang et al. [14]. This work clearly shows that MIMO relay systems offer a higher channel capacities than conventional MIMO systems and cooperative communications. A method to process information in cooperative network is defined as a relay protocol. Many relay protocols including decode-and-forward, amplify-and-forward, compressed-and-forward were proposed in [15]. Channel estimation algorithm of MIMO relay system has been continuously studied [16], [17].

It is obviously seen from the above overview that power allocation, capacity limit, protocols and channel estimation for MIMO relay systems have been well studied [18]. However, a study on Bit Error Rate (BER) which is one of the most important system performance parameters has not yet been reported. In this paper, we concentrate our attention to the BER performance of
MIMO relay systems. Moreover the relay parameters, e.g., position of relays and number of antennas etc., affecting the BER are also investigated in this paper.

The rest of this paper is organized as follows: Section II explains the system model for MIMO relay channels, primary observations are stated in Section III. We present in Section IV the BER results for MIMO relay channel. Finally, concluding remarks are given in Section V.

II. SYSTEM MODEL

We consider in this paper an \(N_t \times N_t\) MIMO system with a single relay node in which decode-and-forward protocol is employed. \(N_t\) is the number of antennas in transmitter and \(N_r\) is the number of antennas in receiver. The system model is illustrated in Fig. 2. Note that every node is equipped with multiple antennas. For this system, the relay node (R) helps the transmission of data from source (S) to destination (D). The distance between S and R (see Fig. 2) is denoted by \(d\) (normalized distance). For example, \(d=0.5\) means that relay node locates in the middle between S and D.

![Fig. 2. System model of MIMO relay channel.](image)

We assume that the relay cannot receive and transmit at the same frequency and the same time. This means that half-duplex (HD) relay mode is considered in this paper. Transmission in Fig. 3 can be divided into two time slots of (normalized) durations \(t\) and \(1−t\). The transmission in half-duplex relay mode can be desended as follows.

In the first time slot \(t\), S transmits data to both R and D. This time slot is called the broadcast (BC) mode.

The second time slot \((1−t)\), is called the multiple-access (MAC) mode. In this mode, D will receive data from both S and R.

Finally, D will recover information from data received from both modes. By doing this, the system is operated with time division half duplex relaying. Fig. 3 graphically show the transmission of both modes respectively. We assume in this paper that S, D and R have the same number of antennas.

![Fig. 3. Half-duplex (HD) relay mode](image)

Throughout this paper, we adopt the following notations. We define \(X_s, X_p, Y_d,\) and \(Y_r\) as the source transmitted data, the relay transmitted data, the destination received data and the relay received data, respectively. The channel coefficient between R and D is denoted by \(h_{RD}\). Other channel coefficients are denoted in the same manner. Subscript 1 and 2 denote to BC mode and MAC mode, respectively. The superscripts indicate the antenna numbers. All channels are assumed to be Rayleigh fading channels. The received signals at the \(1^{st}\) receiver antenna are as follows:

\[
Y_{D1} = \sqrt{\gamma_{SD}} h_{SD}^1 X_s^1 + \cdots + \sqrt{\gamma_{RD}} h_{RD}^1 X_r^1 + n_{D1} \tag{1}
\]

\[
Y_{R1} = \sqrt{\gamma_{SR}} h_{SR}^1 X_s^1 + \cdots + \sqrt{\gamma_{RD}} h_{RD}^1 X_r^1 + n_{R1} \tag{2}
\]

\[
Y_{D2} = \sqrt{\gamma_{SD}^*} h_{SD}^2 X_s^2 + \cdots + \sqrt{\gamma_{RD}^*} h_{RD}^2 X_r^2 + n_{D2} \tag{3}
\]

where \(n_{D1}\), and \(n_{D2}\) are the noises at the destination and relay node receiver in BC mode, respectively and \(n_{D2}\) is the noise at the destination in MAC mode. All the noises are modeled as additive white Gaussian noise (AWGN) with zero-mean and variance \(N_0\).

The channel gain is denoted by \(\gamma\) and the settings are as follows. \(\gamma_{SR} = 1\), \(\gamma_{SD} = 1/d^{\alpha}\), and \(\gamma_{RD} = 1/(1−d^{\alpha})\) where \(\alpha\) is the channel attenuation exponent. \(\alpha=2\) is used in this paper [19]. In this model, the transmitter and the relay node employ BPSK modulation.

The average transmission power, represented by \(P\), is given by [19]

\[
P = P_{S1} + (1−t)(P_{S2} + P_{R2}) \tag{4}
\]

where \(P_{S1}\) is the source transmission power in BC mode, \(P_{S2}\) and \(P_{R2}\) are the source and the relay node transmission powers in MAC mode, respectively.

The definition of signal-to-noise ratio (SNR) is given by

\[
\text{SNR} = 2R E_b \frac{P}{N_0} = \frac{P}{n} \tag{5}
\]

where \(R\) is code rate and \(n\) is noise, \(n=1\) (normalize) was used. Thus, \(E_b/N_0=P/2R\). Note that, for multiple antenna system, \(P\) is the received power for each antenna. The BER results for various values of \(E_b/N_0\) (dB) will be investigated in this paper.

In order to conduct fair comparison experiments, we ensure that the total transmission power of MIMO relay system is equal to that of MIMO system (no relay), single input single output (SISO) relay system, and SISO system (no relay). R and D can extract their received signals by using maximum likelihood detections [20].

III. PRIMARY OBSERVATIONS

Before performing the simulations, five important issues on MIMO relay system are discussed and the primary observations for each topic is described.
A. Effect of Power Allocation to BER
From the literature, the relationship between power allocation, i.e., portion of power in MIMO relay system, and BER performance has not been studied. Regarding the study on MIMO relay capacity, the power allocation is roughly adjusted [14]. For example, \( P_{S1} = P_{S2} = P_{R2} \) (equal power allocation) or \( P_{S1} = 10P_{S2} = P_{R2} \) etc. There are some researches that focus on optimal power allocation [21], [22] but the performance in terms of the BER is not yet reported. If the total power is normalized to be 100 percent. The portions of power used in the simulations are shown in Table I, with these power allocations, we will conduct the simulations to obtain the optimal power allocation in terms of the BER performance.

<table>
<thead>
<tr>
<th>( P_{S1} )</th>
<th>( P_{S2} )</th>
<th>( P_{R2} )</th>
<th>NOTATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>80</td>
<td>-</td>
</tr>
<tr>
<td>40</td>
<td>30</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>60</td>
<td>15</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>50</td>
<td>15</td>
<td>35</td>
<td>-</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td>45</td>
<td>-</td>
</tr>
<tr>
<td>33.33</td>
<td>33.33</td>
<td>33.33</td>
<td>equal power allocation</td>
</tr>
</tbody>
</table>

B. Silent Source in MAC Mode (\( P_{S2}=0 \))
It is known that silent source in MAC mode can simplify the relay transmission [23]. For SISO systems with a single relay node, A. Chakrabarti has concluded that the adoption of silent source in MAC mode (\( P_{S2}=0 \)), can degrade the performance of cooperative communication [24]. We will investigate some cases on power allocation when the source is silent in MAC mode, as listed in Table II.

<table>
<thead>
<tr>
<th>( P_{S1} )</th>
<th>( P_{S2} )</th>
<th>( P_{R2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>75</td>
<td>0</td>
<td>25</td>
</tr>
</tbody>
</table>

IV. RESULTS AND DISCUSSIONS
Simulation results are shown in this section. Regarding the discussions presented below, every parameter is observed at BER of 10^{−9}.
First, the effect of power allocation is considered, as discussed in Section III-A. Fig. 6 shows the BER performances of MIMO relay systems according to various power allocations. For 2x2 MIMO relay systems, it can be seen from the figure that the best power allocation is \( P_{S1}=50, P_{S2}=25 \) and \( P_{R2}=25 \). It is worth to mention that the BER performance strongly depends on the power of S in BC mode. For example, the power of S should be in the range of 40≤\( P_{S1} \)≤60. Comparing with the case of equal power allocation, the power saving obtained from the best power allocation is 1.5 dB. We observe that the best power allocation for 2x2 MIMO relay systems is the same as that for the case of 4x4 MIMO relay systems. We expect that this portion of power is optimal for any number of antennas.

Case I: Relay is placed at the middle between S and D, as shown in Fig. 2.
Case II: The relay node is located closer to the source. This case is shown in Fig. 4. For this case, B. Wang shows that the upper bound and the lower bound of the channel capacity converge [14].
Case III: The relay node is located closer to the destination. The system model of this case is shown in Fig. 5.
The comparison between the best power allocation obtained from the previous empirical experiment for the case of silent source ($P_{S2}=0$) is depicted in Fig. 7. It is obviously seen from the figure that the BER performance of power allocation with silent source is very similar to that of the best power allocation. It is surprising to state that this is different from the case of SISO relay system as discussed in Section III-B. This means that power allocation with silent source, which can simplify the relay protocol, can be applied to MIMO relay system without significant performance degradation.

Regarding the discussions in Section III-C, we next conducted the simulations to show the benefit of having relay nodes in terms of power saving. Fig. 8 illustrates the BER performances of SISO system, MIMO system, SISO relay system and MIMO relay system. From the two rightmost curves, we observe that the employment of single relay in SISO system can reduce the transmission power by about 5 dB. However, it can be seen from the figure that the power saving from the utilization of single relay in MIMO system is more than 8 dB. Therefore, comparing with SISO system, it is worth to have relay in MIMO systems. Furthermore, the performance of 8×8 MIMO system is poorer than 4×4 MIMO relay system. So, the relay node is more suitable for the communication systems that have little space to place large number of antennas.

Next, the worth of having relay node in MIMO system from the information theoretic point of view is considered. Fig. 9 shows the plot of the BER performance against the capacity limit for both 2×2 MIMO system and 2×2 MIMO relay system [14], [25]. The capacity gap for the MIMO system without relay node is about 29 dB while this gap for MIMO relay system is only 24 dB. Therefore, adding a relay node to the 2×2 MIMO system can reduce the capacity gap by approximately 5 dB. Note that the performances shown here are uncoded performance. A reduction in the capacity gap could help any coding scheme to obtain capacity-approaching performance.

As mentioned in Section III-D, the position of relay node does not affect the capacity of MIMO relay systems. However, our next simulation results show that the BER performance is affected by the position of relay node. The number of antennas are varied from 2 to 8. The BER performance for Case I, Case II and Case III are shown in Fig. 10, Fig. 11 and Fig. 12, respectively. The parameters $d$ for Case I, Case II and Case III are 0.5, 0.3 and 0.7, respectively.
between S and D. Therefore, the relay position has a significant effect on the BER performance.

V. CONCLUSIONS

The BER performances for MIMO relay systems are deeply investigated in this work. Our main contributions are as follows. Firstly, we address the optimal power allocation that achieves the best BER performance. Secondly, we have shown that the power allocation with silent source in MAC mode has no effect to the BER performance. Thirdly, we have proved by simulations that it is worth to have relay node in MIMO system. Lastly, we have clearly demonstrated that relay position has an effect to the BER performance. Note that our work can be used as the benchmark for studying the coding scheme for MIMO relay system which could be our future research topic.

TABLE III. THE REQUIRED $E_b/N_0$ (dB) FOR THE THREE CASES OF RELAY POSITION TO ACHIEVE THE BER OF $10^{-6}$

<table>
<thead>
<tr>
<th>MIMO relay</th>
<th>Case I (middle between S and D)</th>
<th>Case II (near S)</th>
<th>Case III (near D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2×2 MIMO relay</td>
<td>21.5</td>
<td>22</td>
<td>22.5</td>
</tr>
<tr>
<td>4×4 MIMO relay</td>
<td>10</td>
<td>11</td>
<td>12.5</td>
</tr>
<tr>
<td>8×8 MIMO relay</td>
<td>5</td>
<td>6.5</td>
<td>8</td>
</tr>
</tbody>
</table>

$E_b/N_0$ (dB) required to achieve BER of $10^{-6}$ are summarized in Table III. From the results, it is clearly seen that the relay node should be put at the middle between S and D. Therefore, the relay position has a significant effect to the BER performance.

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REFERENCES


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