Simple Stabilization Design for Perturbed Time-Delay Systems

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***Abstract*—In this paper, the stabilization design problem for the parametric perturbed systems with a time delay has been addressed. The perturbations are time-varying. Selecting a positive matrix Q and utilizing some linear algebraic techniques, we develop a simple upper solution bound of the Riccati equation and then applying it to the Riccati equation approach, a new condition for testing the stabilizability is proposed. Although the Riccati equation approach is adopted, the obtained stabilizability condition is independent of any Riccati equation and hence can be tested easily. It is shown that this condition is sharper than a previous one. Furthermore, the corresponding stabilization controllers are developed. These controllers are very simple and hence are easy to be implemented. A numerical algorithm is also presented to construct the controllers.**

***Index Terms*—stabilization, time-delay, parametric perturbation, the Riccati equation approach, upper solution bound**

I. Introduction

Time delay exist naturally in physical systems, engineering systems, and so on. When systems possess time delay(s), the eigenvalue number of linear systems would be increased to infinite. This situation might result in unsatisfactory performances or unstable systems. Therefore, time delay is considered as a of instability source of systems. Besides, perturbation will also lead to the eigenvalues of systems cannot be calculated explicitly. Therefore, it is also a source of instability and the control problem of systems then become complicate when time delay(s) and/or perturbations exist. Surveying the literature, researches for the mentioned systems has become an attractive topic over past several decades. In literature, almost of the proposed results involve two topics: (1) the robust stability analysis and (2) the robust stabilization controller design. A number of works have been presented to discuss the above problems during the past decades [1]-[18]. Of those appeared works, the stability analysis problem has been study in [1]-[8] and stabilizability controller design have been developed in [1], [6], [7], [9]-[20]. It often is necessary for the proposed results of controller design to solve different types of LMI. However, in the LMI approach, usually many free matrices in LMI are needed to be determined. This might be a troublesome work. This is the weak point of the LMI approach. Therefore, the objective of this work is to develop state feedback controllers that do not possess any free matrix for perturbed time-delay systems. Extending the approach proposed in [11], we adopt the Riccati equation approach to solve the mentioned problem for the time-delay systems subjected to time-varying perturbations. However, a simple upper matrix bound of the solution of the Riccati equation is first derived by choosing properly the positive definite matrix Q. Then, applying the obtained upper solution bound to the Riccati equation approach, a concise stabilizability condition is presented. This condition do not involve any Riccati equation and hence are easy to be tested. It is also shown that the criterion for system with parametric perturbation is sharper than an existing one. Furthermore, according to the obtained criteria, simple stabilization controllers are developed. These controllers are very simple and hence are easy to be implemented. An algorithm is also proposed to construct these controllers. Furthermore, the obtained results are applied to solve the same problem of the interval time-delay systems. A stabilizability condition and the corresponding controllers are also proposed for the mentioned systems. Finally, we demonstrate the applicability of the present schemes via a numerical example.

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II. Main Results

Consider the time-delay systems with nonlinear perturbations

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A(h,t))x(t)+ \\ &\quad (A_d+\Delta A_d(k,t))x(t-d)+u(t)\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ represents the input, $d>0$ means the delay duration, A, A_d and B denote constant matrices with appropriate dimensions, and $\Delta A(h,t)$ and $\Delta A_d(k,t)$ are time-varying perturbations. Furthermore, it is also assumed that

$$\max_{h\in\Phi} \Delta A(h,t) \le \alpha \text{ and } \max_{k\in\Phi} \Delta A_d(k,t) \le \beta \tag{2}$$

where α and β are positive constants. It is assumed that the pair (A,B) is completely controllable.

In this paper, a simple stabilizability condition and the corresponding state feedback controllers will be developed as follows.

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Theorem 1. If there exists a positive constant η such that

$$\begin{aligned} &A^T + A - 2\eta BB^T + \\ &(2\alpha + \|A_d\| + \beta)I + \\ &\frac{A_d^T A_d + 2\beta\|A_d\|I + \beta^2 I}{\|A_d\| + \beta} < 0 \end{aligned} \quad (3)$$

then the time-delay system 1 with time-varying parametric perturbations can be stabilized by the memoryless state feedback controller

$$u(t) = -\frac{1}{2}B^T Px(t) \quad (4)$$

where the positive definite matrix P satisfies the following Riccati equation

$$A^T P + PA - PBB^T P = -Q \quad (5)$$

with the $n \times n$ real positive definite matrix Q is given as

$$Q \equiv c\left[(2\alpha + \|A_d\| + \beta)I + \frac{A_d^T A_d + 2\beta\|A_d\|I + \beta^2 I}{\|A_d\| + \beta}\right], \quad (6)$$

and c is an arbitrary positive constant.

Proof. Define a positive semi-definite matrix M as

$$M \equiv (P - \eta I)BB^T(P - \eta I), \quad (7)$$

where η is a positive constant. Then, the Riccati equation 5 can be rewritten by

$$\begin{aligned} &(A - \eta BB^T)^T(cI - P) + (cI - P)(A - \eta BB^T) \\ &= -M + Q + \eta^2 BB^T + c(A^T + A - 2\eta BB^T) \end{aligned} \quad (8)$$

By the definition of Q, we have

$$\begin{aligned} &Q + \eta^2 BB^T + c(A^T + A - 2\eta BB^T) \\ &= c\Big[A^T + A - 2\eta BB^T + (2\alpha + \|A_d\| + \beta)I + \\ &\frac{\eta^2}{c}BB^T + \frac{A_d^T A_d + 2\beta\|A_d\|I + \beta^2 I}{\|A_d\| + \beta}\Big] \end{aligned} \quad (9)$$

If the condition 3 is satisfied, then it is obvious that there must exist a constant $c \gg \eta^2 BB^T$ such that the right-hand side of Eq. 9 is negative. Furthermore, the condition 3 can also infer that $A^T + A - 2\eta BB^T < 0$ and we hence can conclude that the matrix $A - \eta BB^T$ is stable. Therefore, equation (8) is a Lyapunov equation and then its solution $cI - P$ is positive definite. This means the solution of the Riccati equation 5 has the following upper bound:

$$P < cI \quad (10)$$

Using the controller 4, the system 1 becomes

$$\begin{aligned} \dot{x}(t) = &(A - \frac{1}{2}BB^T P + \Delta A(k,t))x(t) + \\ &(A_d + \Delta A_d(r,t))x(t-d) \end{aligned} \quad (11)$$

For this system, we construct a Lyapunov function as

$$\begin{aligned} &V(x(t),t) = x^T(t)Px(t) + \\ &c\int_{t-d}^{t} x^T(\tau)\left(\frac{A_d^T A_d + 2\beta\|A_d\|I + \beta^2 I}{\|A_d\| + \beta I}\right)x(\tau) \end{aligned} \quad (12)$$

where the positive definite matrix P satisfies Eq. 5. For convenience, we use symbols V, x, x_d, ΔA, and ΔA_d to replace $V(x(t),t)$, $x(t)$, $x(t-d)$, $\Delta A(h,t)$, and $\Delta A_d(k,t)$, respectively, in the following and later descriptions. Now, taking the derivative along the trajectories of Eq. 1 gives

$$\begin{aligned} \dot{V} = &x^T\Big[A^T P + PA + \Delta A^T P + P\Delta A - PBB^T P + \\ &c\left(\frac{A_d^T A_d + 2\beta\|A_d\|I + \beta^2 I}{\|A_d\| + \beta}\right)\Big]x - \\ &x_d^T c\left(\frac{A_d^T A_d + 2\beta\|A_d\|I + \beta^2 I}{\|A_d\| + \beta}\right)x_d + \\ &x^T P(A_d + \Delta A_d)x_d + x_d^T(A_d^T + \Delta A_d^T)Px + \\ &x^T \Delta APx + x^T P\Delta A^T x. \end{aligned} \quad (13)$$

Since

$$\begin{aligned} &x^T P(A_d + \Delta A_d)x_d + x_d^T(A_d + \Delta A_d)^T Px \\ &\le \frac{1}{\|A_d\| + \beta}x_d^T(A_d + \Delta A_d)^T P(A_d + \Delta A_d)x_d + \\ &(\|A_d\| + \beta)x^T Px \\ &< c[\frac{1}{\|A_d\| + \beta}x_d^T(A_d^T A_d + 2\beta\|A_d\|I + \beta^2 I)x_d + \\ &(\|A_d\| + \beta)x^T x] \end{aligned} \quad (14)$$

$$\begin{aligned} &x^T P\Delta Ax + x^T \Delta A^T Px \\ &\le \frac{1}{\alpha}x^T \Delta A^T P\Delta Ax + \alpha x^T Px < 2c\alpha x^T x \end{aligned} \quad (15)$$

where the bound $P < cI$ is used, then we have

$$\begin{aligned} \dot{V} &< x^T[-Q + c(2\alpha I + \|A_d\|I + \beta I + \\ &\frac{A_d^T A_d + 2\beta\|A_d\|I + \beta^2 I}{\|A_d\| + \beta})]x \\ &= x^T\{-Q + Q\}x = 0 \end{aligned} \quad (16)$$

This shows that the resulting closed-loop system 11 is asymptotically stable if the condition 3 is satisfied. Thus, the proof is completed.

Remark 1. An interesting consequence of this theorem is that the stabilizability condition 3 is independent of the Riccati equation 5. Furthermore, it is also independent of the free variable c.

Remark 2. It is seen that the controller 4 still involves the Riccati equation 5. In the follows, we can simplify the controller design such that the resulting controller is independent of the Riccati equation. This is given as follows.

Theorem 2. Choosing the controller as

$$u(t) = -\eta B^T x(t) \tag{17}$$

If the selected positive constant η such that the stabilizability condition 3 holds, then the perturbed time-delay system 1 can be stabilized by making use of the feedback controller 17.

Proof. From Eq. 17, the closed-loop system now becomes

$$\begin{aligned} \dot{x}(t) = (A - \eta BB^T + \Delta A(h,t))x(t) + \\ (A_d + \Delta A_d(k,t))x(t-d) \end{aligned} \tag{18}$$

Here, we choose the Lyapunov function as

$$\begin{aligned} &V(x(t),t) = x^T(t)x(t) + \\ &\int_{t-d}^{t} x^T(\tau)\left(\frac{A_d^T A_d + 2\beta\|A_d\| I + \beta^2 I}{\|A_d\| + \beta}\right)x(\tau) \end{aligned} \tag{19}$$

This can lead to

$$\begin{aligned} \dot{V} &= x^T[A^T + A + \Delta A^T + \Delta A - 2\eta BB^T + \\ &\quad \frac{A_d^T A_d + 2\beta\|A_d\| I + \beta^2 I}{\|A_d\| + \beta}\Bigg]x - \\ &\quad x_d^T\left(\frac{A_d^T A_d + 2\beta\|A_d\| I + \beta^2 I}{\|A_d\| + \beta}\right)x_d + \\ &\quad x^T(A_d + \Delta A_d)x_d + x_d^T(A_d^T + \Delta A_d^T)x + \\ &\quad x^T \Delta A P x + x^T P \Delta A^{\mathrm{T}} x \\ &\le x^T\Bigg[A^T + A - 2\eta BB^T + \\ &\quad \left(2\alpha I + \|A_d\| I + \frac{A_d^T A_d}{\|A_d\|} + 2\beta I\right)\Bigg]x \\ &< 0 \end{aligned} \tag{20}$$

Therefore, it is seen that if the condition 3 holds, then the perturbed time-delay system 1 can be indeed stabilized by the controller 17. Thus, the proof is completed.

Note that the stabilization controller 17 is very simple. We also give the following algorithm for designing the positive constant η.

Algorithm 1.

Step 1. Set an initial value of $\eta_n = 0$ with $n = 0$.

Step 2. Apply η_n into the condition 3 and check it. If it holds, then stop the algorithm and the controllers 4 or 17 is obtained. Otherwise, go to Step 3.

Step 3. Let $\eta_{n+1} = \eta_n + \xi$, where $\xi > 0$ is an adequate constant. If $\eta_{n+1} > \kappa$ where $\kappa > 0$ is a sufficient large number, then stop this algorithm and the stabilization

Remark 3. In the literature, the choice of the positive matrix Q is still an open problem for those controlled systems by using the Riccati equation approach. The positive matrix Q often is selected as a free matrix or chosen as $Q = cI$ for simplification. For example, by choosing $Q = cI$, the following condition for the perturbed time-delay system 1 is presented by the existed paper Reference [10].

$$\lambda_1(A^T + A - 2\eta BB^T) + 2(\alpha + \|A_d\| + \beta) < 0 \tag{21}$$

Due to the relation $A_d^T A_d \le \lambda_1(A_d^T A_d)I \le \|A_d\|^2 I$ and the inequality $\lambda_1(A+B) \le \lambda_1(A) + \lambda_1(B)$ for $A^T = A$ and $B^T = B$ Reference [21], one obtain

$$\begin{aligned} &A^T + A - 2\eta BB^T + (2\alpha + \|A_d\| + \beta)I + \\ &\frac{A_d^T A_d + 2\beta\|A_d\| I + \beta^2 I}{\|A_d\| + \beta} \\ &\le \lambda_1[A^T + A - 2\eta BB^T + (2\alpha + \|A_d\| + \beta)I + \\ &\frac{A_d^T A_d + 2\beta\|A_d\| I + \beta^2 I}{\|A_d\| + \beta}]I \\ &\le \lambda_1(A^T + A - 2\eta BB^T) + (2\alpha + \|A_d\| + \beta) + \\ &\frac{\|A_d\|^2 + 2\beta\|A_d\| I + \beta^2 I}{\|A_d\| + \beta} \\ &= [\lambda_1(A^T + A - 2\eta BB^T) + 2(\alpha + \|A_d\| + \beta)]I \end{aligned} \tag{22}$$

Therefore, it is obvious that condition 3 is sharper than 21. Besides, using the similar ways, a different stabilizability condition has also been proposed in Reference [11]. However, the tightness between the obtained result and that presented in Reference [11] cannot be compared.

Now, we consider another kind of perturbed time-delay system. The system model is an interval time-delay system as follows.

$$\dot{x}(t) = A_I x(t) + A_{dI} x(t-d) + Bu(t) \tag{23}$$

where A_I and A_{Id} represent interval matrices with appropriate dimensions and have the following properties:

$$A_I = [a_{Ipq}],\ u_{Ipq} \le a_{Ipq} \le v_{Ipq} \tag{24}$$

$$A_{dI} = [a_{dIpq}], u_{dIpq} \le a_{dIpq} \le v_{dIpq} \tag{25}$$

with $p,q = 1,2,\ldots,n$. Here, we define the following matrices:

$$U = [u_{Ipq}], V = [v_{Ipq}], U_d = [u_{dIpq}], V_d = [v_{dIpq}] \tag{26}$$

Define matrices A_i, A_{ij}, and, B_{ij}, respectively, as

$$\bar{A} = \frac{U+V}{2} \text{ and } \bar{A}_d = \frac{U_d + V_d}{2} \tag{27}$$

Then, system (23) can also be represented as follows.

$$\dot{x}(t) = (\bar{A} + \Delta A)x(t) + (\bar{A}_d + \Delta A_d)x(t-d) + Bu(t) \tag{28}$$

where ΔA and ΔA_d now denote the parametric uncertainties with the following properties:

$$|\Delta A| \le R \text{ and } |\Delta A_d| \le S \tag{29}$$

where R and S are defined as

$$R = \frac{V-U}{2} \text{ and } S = \frac{V_d - U_d}{2} \tag{30}$$

Once again, the application of the fact $\|A\| \le \||A|\|$ results in

$$\|\Delta A\| \le \||\Delta A|\| \le \|R\| \equiv \varphi \tag{31}$$

$$\|\Delta A_d\| \le \||\Delta A_d|\| \le \|S\| \equiv \gamma \tag{32}$$

Then, one can apply the results presented in Theorems 1 and 2 to the above time-delay interval systems. Following the similar ways as those of Theorems 1 and 2, we develop the stabilizability condition and the corresponding controllers as follows without proof.

Theorem 3. If there exists a positive constant η such that

$$\bar{A}^T + \bar{A} - 2\eta BB^T + (2\varphi + \|\bar{A}_d\| + \gamma)I + \frac{\bar{A}_d^T \bar{A}_d + 2\gamma\|\bar{A}_d\| I + \gamma^2 I}{\|\bar{A}_d\| + \gamma} < 0 \tag{33}$$

then the time-delay interval system 23 can be stabilized by the following memoryless state feedback controllers

$$u(t) = -\frac{1}{2} B^T P x(t) \tag{34}$$

or

$$u(t) = -\eta B^T x(t) \tag{35}$$

where the positive definite matrix P satisfies the following Riccati equation

$$\bar{A}^T P + P\bar{A} - PBB^T P = -Q \tag{36}$$

with the $n \times n$ real positive definite matrix Q is given as

$$Q \equiv c\left[(2\varphi + \|\bar{A}_d\| + \gamma)I + \frac{\bar{A}_d^T \bar{A}_d + 2\gamma\|\bar{A}_d\| I + \gamma^2 I}{\|\bar{A}_d\| + \gamma}\right] \tag{37}$$

and c is an arbitrary positive constant.

Remark 3. It is seen that, by transferring the interval time-delay system 23 into the perturbed system 28. Then, we can apply directly the results obtained in Theorems 1 and 2 to this interval time-delay system and obtained the result Theorem 3.

III. An Example

To show the merits of the obtained results, we give the following numerical example.

Example 1. This example is given in [10]. Consider the perturbed time-delay system 1 with

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_d = \begin{bmatrix} 0.4 & 0 & 0 \\ -0.2 & 1 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 1 & 0 \end{bmatrix},$$

$$d = 0$$

$$\Delta A = \begin{bmatrix} 0.5\sin t & 0 & 0 \\ 0 & 0 & 0.3\sin 2t \\ 0 & -0.2\sin t & 0.1\cos 2t \end{bmatrix},$$

$$\Delta A_d = \begin{bmatrix} 0.15\cos 3t & 0 & 0 \\ 0.1\sin 2t & 0 & 0 \\ 0 & 0.2\cos 2t & 0 \end{bmatrix}.$$

In [10], it was found that the gain matrix F must satisfy $F \ge 3.65B^T$ such that the closed-loop system can be stabilized. However, using Algorithm 1 and choosing $\xi = 0.1$, we find the algorithm stop at $\eta = 3.3$. This means the gain matrix F can be chosen as $F \ge 3.3$. Obviously, the obtained result is better than that of [10]. Therefore, the memoryless feedback controller now can be designed as

$$u(t) = 3.3B^T x(t) = \begin{bmatrix} 6.6 & 3.3 & 0 \\ 9.9 & 3.3 & 3.3 \end{bmatrix} x(t)$$

Let the states be $x1(t) = -3$, $x2(t) = 4$, and $x3(t) = -5$ for $t \in [-0.5, 0]$. For this case, all simulation results of $x(t) = [x1(t) \quad x2(t)]^T$ and $u(t) = [u1(t) \quad u2(t)]^T$ are shown in Fig. 1 and Fig. 2, respectively. It is seen that all states are regulated to zeros by the proposed controller irrespective of the time-delay and perturbations.

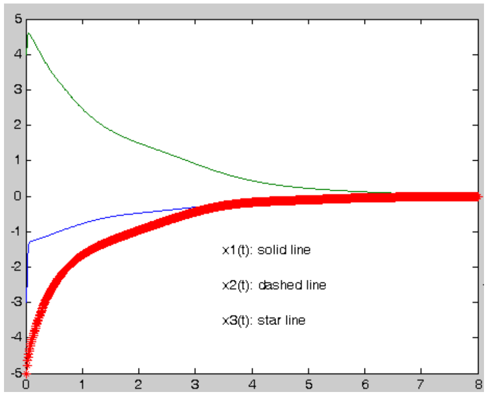

Fig. 1. Trajectory of state $x(t)$ of Example 1

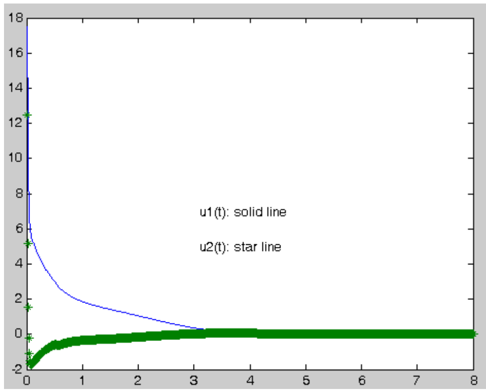

Fig. 2. Trajectory of state $u(t)$ of Example 1

IV. Summary

The stabilization controller design of the perturbed time-delay systems has been solved. The perturbations are time-varying. The author have developed a new stabilizability condition to guarantee the existence of stabilization controllers. This condition does not involve any Riccati equation and hence is easy to be tested. Comparing to an existed result [10], it is shown that the obtained condition is better. Furthermore, another simple stabilization controller that is independent of the Riccati equation has also been developed. An algorithm has also been obtained to show the presented controllers are easy to be implemented. Besides, it is shown that the above results can be applied directly to solve the same problem for the interval time-delay systems. Some similar results have been obtained to construct the stabizability controllers for the mentioned systems. Finally, a numerical example and the corresponding computer simulations have been presented to show the applicability of the proposed schemes.

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