Modeling and Analysis of the Plug-in Electric Vehicles Charging in the Unbalanced Radial Distribution System

Padej Pao-la-or and Banyat Boribun
School of Electrical Engineering, Suranaree University of Technology, Nakhon Ratchasima, Thailand
Email: padej@sut.ac.th; bboribun@windowslive.com

Abstract—This paper presents the modeling and analysis of plug-in electric vehicles (PEVs) charging in an unbalanced radial distribution system. The objective of the study is to evaluate a PEV simulation and the impact of PEV charging on the distribution system. The energy use of a plug-in electric vehicle for a cycle was calculated to plan the charging method of PEVs at a charging station. The PEV charging characteristics were derived from the charging power and queueing analysis. The system power quality according to the load of PEVs charging was analyzed using the unbalanced radial distribution load flow analysis in the Udon Thani Municipality, Thailand. Using this analysis and comparison, the simulation results produced optimal scheduling for charging PEVs improving the unbalance, total power loss, and reducing the number of overloaded feeders.

Index Terms—electric vehicles charging, modeling and simulation, queueing theory, unbalanced radial distribution system

I. INTRODUCTION

CO₂ is the main component of the greenhouse gases that cause global warming. An important source of this gas is the internal combustion engines (ICEs) of vehicles. A survey done in the USA done between 1990-2012 reported that 32% of CO₂ emissions originated from transportation [1]. Thus, the replacement of ICEs with plug-in electric vehicles (PEVs) can reduce CO₂ emissions. The drivers of PEV are motors have no gas emissions. They are independent of fossil fuels because their energy is derived from batteries or electric charging stations. In addition to the various types of the renewable energy sources, PEVs will be the most suitable vehicles for transportation in the future.

An unbalance radial distribution system is an electric system that is closest to the PEV users. Thus, charging PEVs by connecting them to distribution feeders will affect the utility operation and the consumers. The cumulative demand for electricity can exceed the supply of the distribution system during unscheduled charging of PEVs [2]-[5]. This high current supply for the base demand and charging load of PEVs may be higher than normal and cause the degradation of distribution transformers [6], [7]. Voltage drops and power losses are associated with the feeder current and the demand of PEVs charging. The report summarized these problems are more concerned in rural areas than in urban area [8]. The penetration level of PEVs demand affects the power quality indexes such as voltage variation, unbalance, overload of the transformer, and daily load curve [9], [10].

Attention to PEV demand estimation is very important because various distribution system parameters affect PEV charging. Thus, dynamic modeling of PEVs in the first part of this paper describes PEV modeling. Energy requirements were calculated using PEV specifications, routes, and a range of operating conditions. The charging demand was estimated using the remaining capacity of the battery and the queueing theory. The final part of this paper gives a comparison result of dumb and scheduled PEV charging. The evaluation parameters were the unbalanced voltage and current, the total line losses, and the overload current at the feeders.

Fig. 1. The free body diagram of a PEV moving along the road.

II. ELECTRIC VEHICLE MODELING

The amount of charging energy of the PEV depends on the remaining capacity of the battery. The consumed energy of PEV can be determined from a dynamic simulation. The important parameters are position, velocity, acceleration, force, and energy. Fig. 1 presents a PEV with a mass of \( m \) is moving up an inclined road surface at an angle \( \alpha \). The rolling resistance force \( (F_{rr}) \) is caused by the friction between the tyre and the road surface, and it has an associate coefficient of rolling resistance \( (\mu_r) \). The aerodynamic drag force \( (F_{ad}) \) is proportional to air density \( (\rho_{air}) \), drag coefficient \( (C_d) \).
cross-section area of a PEV (A) and the square of its speed. The slope of the road results in a climbing resistance force \( F_{rc} \). The parameters related to this force are the mass of the PEV, the acceleration due to gravity \( g \), and the angle, \( \alpha \), of the slope. The acceleration force \( F_{ac} \) needed by a PEV to reach a distant \( s \) is proportional the acceleration rate, \( a \). A tractive acceleration forces \( F \) is

\[
F = F_{rc} + F_{ac} + F_{cw} + F_{aw}
\]

\[
= m_{mg}g + \frac{1}{2} \rho_{aw} C_{aw} A_s v^2 + mg sin(\alpha) + m \frac{d^2 s}{dt^2}
\]

The mechanical power driving the PEV depends on the conversion efficiency \( \eta_{conv} \) of the battery bank. \( N \) is a discrete order of time steps and the previous time step is \( t_n \). The power supplied by the battery bank at time step, \( t \), is \( P_{BAT}(t) \). The total energy supplied by the battery bank from time 0 to time, \( t \), is

\[
E_{BAT} = \sum_{k=1}^{\infty} P_{BAT}(t_k) + \frac{t \eta_{PEV}}{\eta_{conv}}\sum_{k=1}^{\infty} P_{BAT}(t_k)
\]

It is important to know the total energy consumed in (2) for determining an optimized charging schedule. The difference between \( E_{BAT}(t) \) and remaining capacity of the PEV battery will be considered in selecting the charging method.

III. BATTERY CHARGING MODELING

A model to determine of the active load for a PEV can be derived from the charging current and voltage. The three methods of charging PEV batteries are described in detail as follows [12].

A. Constant Current and Voltage Charging

The preferred method of PEV battery charging starts with constant current charging and finishes with constant voltage charging. The first-time interval \((0, t_c)\) is the constant current charging period. The mode is changed to a constant voltage charging at \( t_c \) until the battery is fully charged at \( t_{ch} \). Suppose \( \tau_1 \) and \( \tau_2 \) are the time constants for charging constant current and voltage respectively, then the total energy transferred to the battery bank of a PEV in the time interval \((0, t_{ch})\) is

\[
E_{ch} = \left[ V_m \tau_2 \left( 1 - e^{-\frac{t_{ch}}{\tau_2}} \right) - V_n \left( \tau_1 - t_c - \tau_2 e^{-\frac{t_{ch}}{\tau_2}} + t_c V_0 \right) \right] I_{n}
\]

The initial voltage of the battery before charging is \( V_0 \). The maximum charging voltage and current are \( V_n \) and \( I_{ch} \), respectively. The constant parameter for battery voltage \( (V_n) \) can be determined from the battery specifications.

B. Constant Current Charging

For this method, the charger supplies a constant current to the battery bank for the time interval \((0, t_{ch})\).

The total energy transferred to the battery bank during this time interval is

\[
E_{ch} = \left[ V_m \tau_1 \left( 1 - e^{-\frac{t_{ch}}{\tau_1}} \right) - t_{ch} \right] I_{n}
\]

C. Constant Voltage Charging

Constant voltage charging starts a constant current, \( I_n \), until a nominal voltage, \( V_n \), is reached. The process continues with constant voltage while the current decreases to zero at time \( t_{ch} \). The total energy transferred to the battery bank for the time interval \((0, t_{ch})\) is

\[
E_{ch} = \left[ -\tau_1 \left( V_m + V_n \right) \left( e^{-\frac{t_{ch}}{\tau_2}} - 1 \right) - \tau V_n \left( e^{-\frac{t_{ch}}{\tau_2}} - e^{-\frac{t_{ch}}{\tau_1}} \right) \right] I_{n}
\]

IV. QUEUEING THEORY FOR PEVS LOAD MODELING

Load flow analysis requires a queueing theory model for the charging load of PEVs [13], [14]. The uncertain parameters of PEVs charging are the arrival time \((t_{arrive})\), charging time period \((\Delta t)\) and charging power rate \((P_{ch})\). For this work, the instantaneous power while a PEV undergoes charging is:

\[
P_{PEV}(t) = P_{max} \left( 1 - e^{-\frac{t}{\tau}} \right) + P_{PEV,0}
\]

According to the battery specifications, \( P_{max} \) is the maximum power capacity, \( P_{PEV,0} \) is the initial capacity, \( t_{max} \) is the maximum charging time, and \( \gamma \) is a constant that can be obtained from the characteristic curve of a battery. At the time required for a battery to become fully charged, \( t_{ch} \), the power of battery can be calculated from (7):

\[
P_{EV,ch} = P_{max} \left( 1 - e^{-\frac{t_{ch}}{\tau}} \right)
\]

Queueing theory is applied because of the random nature of the number of PEVs number and their charging times. According to the M/M/nmax queue, the occupation rate of PEVs on each bus can be calculated from:

\[
\rho = \frac{\lambda}{\mu n_{max}} \approx \frac{1}{n_{max}}
\]

With the M/M/nmax queue, \( M \) is an exponential distribution of incoming PEVs for charging with a mean inter-arrival time of \( 1/\lambda \), mean of service time of \( 1/\mu \), and \( n_{max} \) maximum number of PEVs charged in a parallel charging PEVs at each load bus. If \( n = 0, 1, 2, 3, \ldots, n_{max} \), the probability of \( n \) PEVs being in the charging process at each load bus is

\[
\rho \approx \frac{1}{n_{max}!} \left( 1 - \frac{1}{n_{max}} \sum_{i=0}^{n_{max}} \frac{1}{i!} \right)
\]

Queueing theory is used evaluate the number of PEVs and instantaneous charging loads for load flow analysis. The process of modeling can be described as follows.

1. A random number over the interval $[0, 1]$ is generated using a uniform distribution.
2. A maximum number ($n_{\text{max}}$) of PEVs is determined based on the rated capacity and remaining energy of all PEV batteries.
3. The lowest $n$ in the set $[0, 1, 2, \cdots, n_{\text{max}}]$ is determined that satisfies the inequality
   \[ r[0,1] < p(n) \] (10)
4. Set $t_{\text{max}}$ is the maximum time to charge a PEV battery, the time period charging for the PEVs for $i = 1, 2, 3, \cdots, n$ can be calculated from the above conditions. If $r(0,1) < \mu_{n_{\text{max}}} e^{-r_{n_{\text{max}}} t_{\text{max}}}$, the charging time ($t_{i}$) is $t_{\text{max}}$, otherwise the charging time can be calculated from $t_{i} = -(1/\mu_{n_{\text{max}}}) \ln( r(0,1)/\mu_{n_{\text{max}}})$.
5. The total active power charging PEVs ($P_D$) is determined using the following stochastic formula:
   \[ P_D = P_{\text{PEV max}} \left( n - \sum_{i=1}^{n_{\text{max}}} e^{-r_{n_{\text{max}}} t_{\text{max}}} \right) \] (11)

The total active power in (11) is the average power of the PEV demand on the unbalanced radial distribution system. The most impact resulting from the PEV demand estimation is the maximum number of PEVs that can be charged in parallel ($n_{\text{max}}$). According to the specification of PEVs and remaining capacity of the battery, these models are evaluated to estimate $P_D$ and input in the load flow analysis.

V. UNBALANCED RADIAL DISTRIBUTION LOAD FLOW ANALYSIS

The unbalanced radial distribution load flow analysis has some specific characteristics. These include a transformer, an untransposed line, and unbalanced loads along the feeder with single-phase and two-phase laterals. The accuracy of the load flow analysis results depends on the modeling of system components. The details of the topology and modeling of the unbalanced radial distribution components are described as follows.

A. Feeder Line Modeling

The models of a three-phase, two-phase, or single-phase overhead or underground feeder line link have a sending bus, $n$, and a receiving bus, $m$. The impedance value may be zero, in the case of two-phase or single-phase systems. The modeling equations describe the relationship between voltages and currents of the sending (node $n$) voltages and the receiving ends (node $m$). The line currents and the line-to-ground voltages modeling the feeder line are as follows [15]:

\[
\begin{align*}
[\text{VLG}_{abc}]_{n} &= [a] \cdot [\text{VLG}_{abc}]_{n} + [b] \cdot [I_{abc}]_{n} \\
[I_{abc}]_{n} &= [c] \cdot [\text{VLG}_{abc}]_{n} + [d] \cdot [I_{abc}]_{n}
\end{align*}
\] (12) (13)

The actual data available for the feeder will be the positive and zero sequence impedances. An approximate line model can be obtained by applying the reverse impedance transformation from symmetrical component theory. In the symmetrical component term, the modeling equation for voltages and currents at nodes $n$ and $m$ is

\[
[\text{VLG}_{abc}]_{n} = [\text{VLG}_{abc}]_{n} + [Z_{\text{approx}}] \cdot [I_{abc}]_{n}
\] (14)

B. Load Modeling

The types of loads in the unbalanced radial distribution system are spot and distributed loads. A spot load can be connected with wye or delta topology. The updated voltage ($\{V_{abc}\}$) in each iteration of the load flow calculation must be changed from the nominal voltage ($\{V_{0,abc}\}$) until it reaches a convergent value. The current load injection for the specified complex powers of the wye-connected and delta-connected load [15] is given by:

\[
\begin{align*}
[\text{IL}_{abc}]_{n} &= \left( S_{0,a} \cdot \left( \frac{|V_{n,a}|}{|V_{n,a}|} \right) + \frac{1}{jB_{n,a}} \right) \\
&= \left( S_{0,a} \cdot \left( \frac{|V_{n,a}|}{|V_{n,a}|} \right) + \frac{1}{jB_{n,a}} \right) \\
\end{align*}
\] (15)

The constant, $n$, of the three load types is equal to 0, 1, and 2 for constant power, constant current and constant impedance, respectively. The distributed load can be modeled as being uniformly distributed along a feeder connected buses $n$ and $m$. The total line distributed load can be lumped in the two-end bus of the feeder as described in Eq. (16)

\[
[S_{abc,n}] = \frac{1}{2} [P_{abc} + jQ_{abc}]
\] (16)

C. Shunt Capacitor Modeling

The merits of shunt capacitor banks in the distribution systems are voltage regulation and provision of reactive power support. The first step of the shunt capacitor modeling is the susceptance calculation. The line currents for the three-phase shunt capacitor are given by:

\[
\begin{align*}
IC_{a} &= \left[ \begin{array}{c}
jb_{b}\text{V}_{cn} \\
jb_{b}\text{V}_{cn} \\
jb_{b}\text{V}_{cn}
\end{array} \right] \\
IC_{b} &= \left[ \begin{array}{c}
1 \\
-1 \\
0
\end{array} \right] \left[ \begin{array}{c}
jb_{b}\text{V}_{ab} \\
jb_{b}\text{V}_{ab} \\
jb_{b}\text{V}_{ab}
\end{array} \right] \\
IC_{c} &= \left[ \begin{array}{c}
0 \\
1 \\
-1
\end{array} \right] \left[ \begin{array}{c}
jb_{b}\text{V}_{ab} \\
jb_{b}\text{V}_{ab} \\
jb_{b}\text{V}_{ab}
\end{array} \right]
\end{align*}
\] (17)
D. Transformer Modeling

The transformer modeling used in load flow analysis is generalized for the connections so that it has the same form as was developed for feeder line modeling. In the forward sweep calculation, the voltages at bus \( m \) ([\( VLN_{abc} \)]) are defined as a function of the voltages at bus \( n \) ([\( VLN_{ABC} \)]) and the currents at bus \( m \) ([\( I_{abc} \)]). The modeled equation is,

\[
[VLN_{abc}] = [A] [VLN_{ABC}] - [B] [I_{abc}] \tag{18}
\]

In the backward sweep calculation, the equations for computing the voltages and currents at bus \( n \) as a function of the voltages and currents at bus \( m \) are given by (19):

\[
[VLN_{abc}] = [A] [VLN_{ABC}] - [B] [I_{abc}] + [c] [I_{abc}] + [d] [I_{abc}] \tag{19}
\]

The constant \([A], [B], [a], [b], [c], \) and \([d] \) depend on the type of transformer connection. The details of the modeling equations have been described [12].

E. Load Flow Algorithm

The first step of unbalanced radial load flow analysis is the data preparation. The data for all components is synthesized and modeling for load flow calculation is developed. The details of the algorithm have been described [16], [17].

VI. SIMULATION RESULTS

The first part of the work is a dynamic simulation calculating energy use of PEVs. The restoration of the full state of charge (SOC) of a PEV’s battery can be made by the charging it using a suitable method. In the second part of this work, the PEV demand was estimated using queueing theory. Finally, the third part of the work is the charging simulation of the PEVs with two scenarios, dumb and optimal scheduling. According to the simulation results, the difference of the power quality parameters was verified and the importance of an optimal schedule of PEVs charging is discussed.

A. Dynamic Simulation

The test PEVs were the BYD: K9 and EMOSS: CM Mission 150e. The locations and distances of the routes are detailed in Table I. The parameters of the simulation are detailed in Table II. The locations of the charging stations are A and D. The acceleration distance from a full stop to a maximum speed is 100 m. The simulation time interval is 0 s to 1354.9 s and the time-steps are 1 ms. The tractive force and supplied energy of the batteries are shown in Fig. 2 (a) and Fig. 2 (b), respectively. The efficiency of energy transfer from a battery to a PEV was set at 85%. According to the trapezoidal rule calculation, the energy use of the BYD: K9 and EMOSS: CM Mission 150e was 72.52 kW·h and 20.51 kW·h, respectively. The remaining capacity of the batteries of the two PEVs was 251.48 kW·h and 31.48 kW·h, respectively. The charging simulation considered only the BYD: K9 because it consumes more energy than the EMOSS: CM Mission 150e. The state of charge and voltage of the battery before charging were equal to 40% and 365 V, respectively. According to the rated capacity of the battery, the required energy for charging is equal to 324–251.48=72.52 kWh. This is the maximum PEV demand on the test system.

| Table I: Location and Distance of the Route |
|-----------------|-----------------|-----------------|-----------------|
|                | Outward Route   | Inward Route    |                |
| Distance (m)   | Distance (m)    | Distance (m)    |                |
| A B             | C D             | E F             |                |
| 950             | 2,550           | 3,150           |                |
| B C             | 2,950           | C B             | 2,550           |
| C D             | 2,950           | B F             | 1,950           |
|                 |                 | F A             | 1,000           |

<table>
<thead>
<tr>
<th>Table II: Simulation Parameters of the PEVs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification</td>
</tr>
<tr>
<td>Coefficient of rolling resistance (( \mu_s ))</td>
</tr>
<tr>
<td>Air density (( \rho ), kg·m(^{-3}))</td>
</tr>
<tr>
<td>Aerodynamic drag coefficient</td>
</tr>
<tr>
<td>Bus frontal area (( A ), m(^{2}))</td>
</tr>
<tr>
<td>Overall mass of the bus (( m ), kg)</td>
</tr>
<tr>
<td>Slope angle of the road (( \alpha ), degree)</td>
</tr>
</tbody>
</table>

(a) Tractive force

(b) Supplied energy of batteries

Fig. 2. Dynamic simulation results.

B. Queueing Theory and PEVs Demand Estimation

The dynamic simulation showed that the maximum demand of a PEV was equal to 72.52 kW·h. The number and charging time of PEVs depends on the customer decisions, SOC of the battery, the number of PEVs on the road, and the charging price. The charging simulation applied queueing theory estimates of the PEV loads for load flow analysis to assess the impact of PEVs charging. In such scenarios, we might model the arrival of PEVs as a nonhomogeneous Poisson process. For the \( M/M/20 \) queueing model, the maximum capacity of a charging station is \( n_{max}=20 \) parallel charging PEVs at each load bus. According to the driving cycle and the route of this study, the mean rate of charging of the station is equal to \( \mu = 10 \). With the calculation of \( 10^{3} \) samples, the stochastic real
power demand and basic load of the n-PEV system are shown in Fig. 3 (a) and Fig. 3 (b), respectively. According to the basic load, the average PEV demand is approximately 50 kW.

1) Minimize \( |V_d/V_1| \) subject to \( V_d(t)/V_1(t) \leq |V_d/V_1|_{\text{avg}} \).
2) Minimize \( |I_2/I_1| \) subject to \( I_2(t)/I_1(t) \leq |I_2/I_1|_{\text{avg}} \).
3) Minimize \( P_{\text{loss}} \) subject to \( P_{\text{loss}}(t) \leq P_{\text{loss,avg}} \).
4) Minimize \( OL \) subject to \( OL(t) \leq OL_{\text{avg}} \).

Comparison of the results of dumb charging PEVs with optimal charging shown in Fig. 4 and Fig. 5. The two charging simulations are described as follows.

- Charging for each 1 hour in the daytime and 2.5 hours in the evening.
- Optimal charging for a specified time period conforming to the optimal constraints. The initial value for the average charging power for constant current and voltage charging was set to 100 kW. The charging power, starting time, and the time for charging were varied according to the optimal constraints.

The impacts of PEV demand on the unbalanced radial distribution system were different for dumb and optimal charging. The unbalance voltage level of bus-42 in Fig. 4(a) showed that dumb charging improves unbalance problem. This result may be due to the PEVs load being connected to the unbalanced phase. Alternatively, unscheduled charging, as shown in Fig. 4(b), caused a more unbalanced current level at branch-60. The total power loss of system was proportional to variance in the current along the feeder.

C. Charging Simulation

The system data of the study was obtained from the unbalanced radial distribution system of Provincial Electricity Authority (PEA), Udon Thani province. The selected feeder is the 4th circuit of the Non Sung service station along Thaharn Road, in the Muang District, Udon Thani. The charging PEVs were connected to system buses 108 and 169. The maximum load of PEVs was 145kW or the 3.37\% of the maximum base load 4.3MW. The parameters of interest in the current study are the unbalanced of voltage and current, total line losses, and number overloaded of feeders. PEV charging was simulated with 2 scenarios, i.e., dumb charging and optimal charging. For dumb charging PEVs without services, the simulation was tested under two circumstances: 1) charging for 1 hour during the daytime and 2) charging for 2.5 hours in the evening. The power quality parameters were assessed at the bus-42 and branch-60 because these are the most distant from the substation.

For the optimal scheduling of PEV charging, the charging hours \((t_k)\) were assigned as 1st, 2nd, 3rd, ..., 24th. The parameter \(X(t_k)\) is the value of voltage or current at the charging time \(t_k\). The average value is indicated by \(X_{\text{avg}}\). The total loss power \((P_{\text{loss}})\) and number of overload feeders \((OL)\) are constrained for control of PEV charging. The objective functions and constraints of the optimal scheduling of the PEVs charging for this work are as follows.

The increment of total power loss because of the dumb charging was more than the optimal charging as shown in Fig. 5(a). The last parameter assessed in this work is the number of overload feeders, as shown in Fig. 5(b). The 1st to 10th hour is shown the overload feeder numbers of an optimal charging more than a dumb charging method. After this time interval, the adjacent 1 hour charging method and evening 2.5 hour charging method causing overload in the number of feeders more than optimal charging for the most time interval.
charging schedule. An accurate estimation of PEV demand and an optimal charging schedule is necessary to prevent issues such as power quality degradation, increased line losses, and feeder overload. The presence of PEVs in the distribution system requires different results than the optimal schedule. Thus, the charging simulation showed the level of these parameters related to the PEV charging load had significant effects. The charging simulation showed the level of these parameters related to the PEVs charging demand. Unscheduled PEV charging caused problems with power quality, but different results than the optimal schedule. Thus, the presence of PEVs in the distribution system requires accurate estimation of PEV demand and an optimal charging schedule.

ACKNOWLEDGMENT

This work was supported in part by a grant from School of Electrical Engineering, Institute of Engineering, Suranaree University of Technology.

REFERENCES


Padej Pao-la-or received B. Eng. (1998), M. Eng. (2002) and D. Eng. (2006) in Electrical Engineering from Suranaree University of Technology, Thailand. He is an Associate Professor at the School of Electrical Engineering, Institute of Engineering, Suranaree University of Technology, Nakhon Ratchasima, Thailand. His fields of research interest include a broad range of power systems, finite analysis, optimization and artificial intelligence, electromagnetic field, electrical machinery and energy conversion. He has joined the school since December 2005 and is currently a member in Power System Research, Suranaree University of Technology.

Banvat Boribun was born in Ros Et, Thailand in 1975. He received B. Eng. (1998), and M. Eng. (2005) in Electrical Engineering from Suranaree University of Technology, Thailand. He is the Ph.D. student at the School of Electrical Engineering, Institute of Engineering, Suranaree University of Technology. His fields of research interest include a broad range of power systems and optimization.