The Separation and Combination Method for Designing Piecewise-Adaptive Automatic Control Systems

Invited Paper

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Abstract—The paper proposes a method for creating of adaptive automatic control systems based on the method of task separation and combination of solutions proposed by the authors. At the first stage, the task of designing an adaptive system is divided into subtasks, which are solved using robust regulators. At the final stage, decisions can be partially combined or simplified by gluing solutions that are not significantly different, i.e. applying the same solutions in related situations. With the help of numerical modeling and optimization, the effectiveness of the proposed method has been demonstrated using an illustrative example. The method greatly simplifies the task of designing adaptive systems and their implementation.

Index Terms—automation, regulators, controllers, numerical optimization, feedback, adaptive systems, self-tuning systems, robust regulators

I. INTRODUCTION

Feedback control is rather relevant field of researches; it is the only way of precise control of any technical device or setup [1]–[4]. The newest problems are involved with the control of complex objects having sophisticated mathematical model. Regulator for such objects cannot be calculated by analytical methods, but numerical optimization has provided the sufficient tool for this task [1]–[4]. But real progress in controlling stationary objects is different from the situation with nonstationary objects. It can be resolved with adaptive regulator only [5]–[27]. This problem is still resolved mostly theoretically in the common recommendations, but practical results in this field are seldom. The paper proposes a new method for control of nonstationary object, which is much more simple then any of the known ones.

The proposed method is based on combining two mutually exclusive approaches to the solution of any task, namely the fragmentation method and the method of consolidation of several tasks into one.

The division of the task is carried out in the case when one performer cannot cope with the task completely. In this case, it is desirable to divide the problem into two subtasks so that each of them is sufficiently independent and can be solved separately. If, as a result of the division, the subtasks obtained remain unfulfilled by those executors or executing (or computing) devices, then the fragmentation should be continued until the subtasks obtained are simple enough to be solved by these performers.

Gluing involves assigning several tasks to one performer on the basis that one task assigned to him is too simple; he has a redundant resource (computational, energy, or other). Gluing tasks may also be appropriate in many cases.

Humanity has developed both approaches intuitively. On the first approach, the division of labor is based, on the second approach, cooperation in which one performer or consumer unites their capabilities or needs, which reduces the cost of a resource per unit of operation.

The proposed approach can be effective in developing adaptive systems, as well as in creating intelligent control systems that analyze a lot of information and make many different decisions based on this analysis.

This paper focuses on the application of this approach to the creation of adaptive control systems.

II. THE PROBLEM STATEMENT

Control of an object that changes its parameters during operation can be solved using robust regulators that are insensitive to such changes (if they are small) or using adaptive regulators (if the changes are extremely large). Each of these solutions has many drawbacks.

Robust regulators implement the gluing approach. Indeed, the same regulator is proposed to solve the control problem for a whole variety of variants of the object model. Here gluing takes place not in the form of a parallel solution of several tasks, but in the form of a unique solution for all tasks from this cluster, i.e. subclass.

Adaptive regulators implement the crushing approach. Indeed, if it is impossible to find a model of a regulator
that would provide control of an object in the entire space of values of object parameters, then this space can be divided, for example, into two subspaces for each of the parameters. If there are only two parameters, then the partition of the task will be in two subtasks, if there are more parameters, for example, $N$, then for each parameter the space is divided into two parts, the number of parts is multiplied by two; therefore there will be $2^N$ of such clusters.

Thus, at the junction of these two opposite approaches, piecewise-adaptive systems can be proposed, which are robust within small changes of object parameters and adaptive as a whole.

At the same time, it is necessary to talk about gluing, because without it, the number of clusters obtained can be so large that the proposed approach will not differ much from the adaptive control method.

The advantages of such a solution are that it simplifies both tasks, which, as a rule, need to be solved when developing adaptive systems, namely, the task of determining the current object model and the task of calculating the regulator for the current object model. Instead, the task comes down to defining one of the pre-defined clusters to which the current model parameters relate, and using a pre-calculated controller (regulator) that is robust for any set of model parameters related to this cluster. This task requires a painstaking preliminary analysis of the influence of model coefficients on stability and other properties of the system, as well as splitting the parameter space into cluster subspaces, while simultaneously calculating regulators for each cluster individually.

The paper poses the problem of studying this approach on a case in point.

### III. Robust System as a Prototype of an Adaptive System

The main idea of the robust approach is to ensure the stability of the locked systems not only with an object with the given parameters values, but also for all variants of the values of these parameters within their allowable ranges of variation, using the same regulators [28].

Of course, it is impossible to provide a solution to this problem using the optimization method for modeling for all intermediate values of these parameters. Therefore, a choice of representative models of the object is needed, with which further optimization is carried out.

The most obvious, but not always the right way is to choose the border values for each parameter of the object. The insufficiency of this approach is that the combination of some average values of the parameters may turn out to be less favorable for control than the border values. Therefore, in this approach, the not sufficiently substantiated hypothesis is laid down that the situation between the border values of the parameters changes smoothly, without special situations.

For reliability, one should check how true this assumption is, for example, by choosing intermediate values of parameters and checking properties of the resulting system. It can be done by numerical simulation. This question requires a separate study.

### IV. Robust System Calculation Method

After selecting of the parameters of the object model, it is necessary to carry out parallel modeling of as many systems as is the number of the selected sets of these parameters [28].

For example, if there is only one parameter, then it may be sufficient to use two object models. If there are two parameters, then the minimum number of models is four, and so on.

When the regulator is numerically optimized for a single object model, the integral criterion of system quality $\Psi(e, T)$ is used as the optimality criterion. It depends on the error in the control loop, denoted by $e(t)$ [29]–[33]. Here $T$ is the simulation and integration time. For example,

$$\Psi(e, T) = \int_0^T |e(t)| dt$$

In the numerical optimization of a single regulator for $N$ objects, $N$ systems should be modeled, each of which consists of a different object and the same regulator, i.e., it is necessary to simulate $N$ identical regulators and $N$ different objects pairwise integrated into the system. As an optimality criterion, it is necessary to use the sum of individual criteria. In this case, in each $i$th system there will be an error $e_i(t)$. In this case, the general integral criterion will be equal to the sum of particular criteria for all systems:

$$\Psi_{\text{tot}}(e, T) = \sum_{i=1}^{N} \Psi_i(e, T).$$

To ensure the required quality of the system with different values of the parameters of the object in [5] it was proposed to use a set of objects described by the model of the regulator, each of which is characterized with different values of parameters, as shown in Fig. 1.

![Regulator optimization scheme](image)

**Fig. 1.** Regulator optimization scheme: 1, 2 – object models with different values of parameters; 3, 4 – identical models of regulators; 5 – test exposure driver; 6 – systems quality analyzer; 7 – regulator parameters optimizer

For each of these objects, an identical regulator is simulated. Its parameters are calculated by the method of optimization by the criterion, which includes the sum of errors of all systems of the set.

The features of the robust regulator are that it must provide stable control with acceptable quality, provided that the parameters of the object model are changed, or are not known accurately. Moreover, this success of control is achieved not due to changes in the model of the
regulator, but due to finding such a universal model of it, which would provide a solution to the problem with any possible combinations of the parameters of the object model. It is obvious that the solution of the problem of successful control may be unattainable by this method.

V. ADAPTIVE SYSTEM

Unlike robust regulators, adaptive regulators can change the parameters of their mathematical model depending on the current parameters of the object model. The class of tasks that can be solved in this way is much wider, and the results can be significantly better. But designing of adaptive regulators is much more difficult than designing of robust regulators. The main difficulty in implementing adaptive systems consists, firstly, in determining the current object model, and secondly, in calculating the best regulator for this current model.

Simplification of the method for solving this problem can be achieved by splitting the variants of possible mathematical models of the object into a countable set and using the robust control method within each of these sets. In this case, the each concrete subtask of the robust control is simplified compared to the attempt to provide the required properties of the system using a single robust regulator.

On the other hand, detailed identification of all object parameters in this case is no longer necessary, since it is sufficient to ensure only recognition of the characteristic features of the object model, sufficient to assign the current model to one of the previously selected clusters.

Let, for example, the control object has a mathematical model in the form of the transfer function \( W_0(s) \), as following,

\[
W_0(s) = \frac{k}{(Ts + 1)^n} \exp\{-s\tau\} \tag{1}
\]

The non-stationary properties of the object consist in the fact that in some predetermined limits, all the parameters of its model included in this function can change, namely: \( k \) is gain factor, \( T \) is time constant, \( n \) is model order and \( \tau \) is delay link time constant.

The output signal of the object \( Y(t) \) should coincide as accurately as possible with the prescription \( V(t) \), the object is affected by unknown disturbance, and the parameters of the object model are slow changing in an unknown way in time (that is, these changing is 100–1000 times slower than the rate of change of the output signals of the object).

If the parameters of the transfer function (1) change over time, the robust regulator remains unchanged, while the adaptive regulator should change depending on these changes:

\[
W_k(s) = W_k(s, n, k, T, \tau) \tag{2}
\]

In our concept, the piecewise-robust regulator is a regulator whose mathematical model depends on one parameter which is the number of the subset to which the current state of the object model is assigned. In the case of a piecewise-robust regulator, all the coefficients of the regulator can be taken from a pre-calculated table, that is, be fixed from a predetermined subset. This is much simpler than the continuous calculation of new coefficients, based on the set of newly defined parameters of the object model.

Determining the situation of assigning a model to one of the predefined subset is also much easier than determining the entire object model completely. As a result, two complex procedures (identification of the object model and calculation of the regulator model) are replaced by two simple procedures (assignment of the object model to the selected subset and selection of the regulator model depending on this subset).

VI. EXAMPLE OF SPLITTING A SET OF OBJECT PARAMETERS INTO SUBSETS

One of the primitive variants of dividing the set of parameters of an object into subsets is to split the region of admissible values of each of the changing parameters.

Let in (1) \( n = 2, T = 1, 1 \leq k \leq 2, 0.2 \leq \tau \leq 0.3 \).

For simplicity, developer can apply the division of intervals into two, if there are no valid reasons for another choice. If, as a result of solving the problem, it turns out that such a splitting is not enough, it will be necessary to apply finer fragmentation of intervals according to one or more of the selected parameters.

When using the border values of the intervals of change of non-stationary parameters, we obtain four different models of the object.

\[
W_1(s) = \frac{1}{(Ts + 1)^2} \exp\{-2s\},
\]

\[
W_2(s) = \frac{1}{(Ts + 1)^2} \exp\{-3s\},
\]

\[
W_3(s) = \frac{2}{(Ts + 1)^2} \exp\{-2s\},
\]

\[
W_4(s) = \frac{2}{(Ts + 1)^2} \exp\{-3s\}. \tag{3}
\]

It is necessary to create a unit project with all these object models and with the equal regulators, the coefficients of which are set from the optimization block. In each of the systems there will be a control error, respectively, \( e_1(t), e_2(t), e_3(t), e_4(t) \). The cost function will be calculated as a time integral of the sum of the modules of these errors, multiplied by the time from the beginning of the transient process:

\[
\Psi_1(T) = \frac{1}{T} \sum_{i=1}^{4} |e_i(t)| dt \tag{4}
\]

Fig. 2 shows a project in the software VisSim for modeling and optimizing the PID-regulator for four objects simultaneously defined by transfer functions (3) with the helps of the cost function (4).

This software VisSim has been created by the corporation Visual Solution Inc. (USA). The main developer of the software and the Head of the corporation is Peter Darnell. Along with the system itself, a number of packages for its expansion have been released.
significantly increasing the already tangible capabilities of this system [34].

The obtained coefficients of the PID-regulator are shown at the output of the "parameters unknown" blocks. Transient processes in all four systems are shown on the oscilloscope on the right. Only in one case the transient process can be considered satisfactory, this refers to the object with the transfer function $W_0(s)$. In other cases, the overshoot is more than 20%, in the case of $W_4(s)$ it is 60%. In this case, the problem is not solved by a single robust regulator. Therefore, it is proposed to split the interval of change of the object's gains into two subset. We get two subtasks, identical to the previously set task, for the following two sets of intervals of varying parameters:

$$Q_1: 1 \leq k \leq 1.4, \ 0.2 \leq \tau \leq 0.3,$$

$$Q_2: 1.4 \leq k \leq 2, \ 0.2 \leq \tau \leq 0.3.$$

The result of optimization according to a similar scheme for the first subtask does not give the desired solution; in one of the cases the overshoot exceeds 40%. Therefore, we also implemented the division of the interval in the time constant of the delay link into two subintervals, the task is converted into four different subtasks for the following subsets of coefficients:

$$Q_3: 4.1 \leq k \leq 1.19, \ 1.97 \leq k \leq 1.4$$

$$Q_4: 1 \leq k \leq 1.4, \ 0.25 \leq \tau \leq 0.3$$

$$Q_5: 1.4 \leq k \leq 2, \ 0.25 \leq \tau \leq 0.3.$$

This task for the specified action algorithm also does not provide the desired solution.

Further, the gain ranges can also be further divided into two subintervals each: $1 \leq k \leq 1.19$ , $1.19 \leq k \leq 1.4$ , $1.4 \leq k \leq 1.66$ and $1.66 \leq k \leq 2$ . The solution of the problem according to the considered algorithm also does not lead to the desired result, since only in one case for the obtained interval it turns out to calculate a robust regulator that gives satisfactory transient process with a small overshoot of not more than 10%.

It is therefore proposed to modify the cost function so that it more effectively provides suppression of overshoot. To this end, the cost function is introduced into an error growth detector according to the following relationship [4]:

$$\Psi(x) = \sum_{l=0}^{t} |e_l(t)| + \sum_{l=0}^{t} F[e_l(t)]$$

Here the function $F[x]$ means the positive part of the expression in square brackets:

$$F[x] = \max\{ x,0 \}$$

$\Psi(x)$ is calculated at $x$ the point on the time axis before the moment of the next transient process.

This process is performed at each point of the time axis, including it is necessary to evaluate the overshoot in the case of the previous transient processes.

As a result, the final division of the intervals by the time constant of the delay link into two subintervals, and

![Fig. 2. A project in the VisSim program for modeling and optimizing the PID-regulator for four objects defined by transfer functions (3) by value function (4)](image)

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![Fig. 3. Scheme for calculating the updated cost function from relations (5), (6)](image)

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the division of the gain intervals into four subintervals, gives eight different intervals of the object parameters for which robust regulator can be found. Table I shows the values of the PID-regulator coefficients depending on the interval at \(0.2 \leq \tau \leq 0.25\), where in the column for the coefficient the average value of this coefficient is given. Table II gives the values of the PID-regulator coefficients at \(0.25 \leq \tau \leq 0.3\), here additional dividing of the last interval became necessary.

Thus, nine fixed sets of values for the PID-regulator were obtained.

**Table I: The Coefficients of the PID-Regulator Depending on the Average Value of the Coefficient of the Object in the Range \(0.2 \leq \tau \leq 0.25\)**

<table>
<thead>
<tr>
<th>(k)</th>
<th>(K_P)</th>
<th>(K_I)</th>
<th>(K_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.095</td>
<td>4.45</td>
<td>1.2</td>
<td>1.86</td>
</tr>
<tr>
<td>1.295</td>
<td>4.52</td>
<td>1.21</td>
<td>1.53</td>
</tr>
<tr>
<td>1.535</td>
<td>3.61</td>
<td>0.864</td>
<td>1.3</td>
</tr>
<tr>
<td>1.835</td>
<td>2.98</td>
<td>0.864</td>
<td>1.54</td>
</tr>
</tbody>
</table>

**Table II: The Coefficients of the PID-Regulator Depending on the Average Value of the Coefficient of the Object in the Range \(0.25 \leq \tau \leq 0.3\)**

<table>
<thead>
<tr>
<th>(k)</th>
<th>(K_P)</th>
<th>(K_I)</th>
<th>(K_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.095</td>
<td>2.7</td>
<td>1.07</td>
<td>0.89</td>
</tr>
<tr>
<td>1.295</td>
<td>2.8</td>
<td>1.03</td>
<td>0.88</td>
</tr>
<tr>
<td>1.535</td>
<td>2.9</td>
<td>0.95</td>
<td>0.88</td>
</tr>
<tr>
<td>1.7</td>
<td>2.8</td>
<td>0.74</td>
<td>1.2</td>
</tr>
<tr>
<td>1.82</td>
<td>2.87</td>
<td>0.64</td>
<td>1.3</td>
</tr>
</tbody>
</table>

The obtained result can be additionally used to create not piecewise adaptive systems, but adaptive systems in traditional terminology. To this end, Fig. 4 shows smoothed graphs of the coefficients for different values of the object's gain and the delay time constant in it. Smoothing is done in MS Excel. The simulation has showed that the intermediate values of the sets of coefficients of the regulators calculated from these graphs correspond well enough with the object control problem. In all cases, the overshoot does not exceed 10–15%.

**VII. CONSOLIDATION OF RESULTS**

It can be noted that changes in the coefficient of the integral link of the regulator in all cases is insignificant. It can be taken equal to \(K_I = 0.8\) to eliminate the influence of this coefficient on the overshoot. It is obvious that the coefficient of the proportional link varies to the greatest extent; therefore, it provides the adaptive properties of the system to the greatest extent. The need to change the coefficient of the derivative link can be investigated by additional modeling, but the concept of a piecewise adaptive regulator does not require this. According to the results of additional modeling, it should be noted that in the third row of Table II it is desirable to reduce the coefficient to the value \(K_P = 2.6\). Fig. 5 shows transient processes \(Y(t)\) for objects with object and regulator parameters in Table I, with \(K_I = 1\) for all cases. Fig. 6 shows transient processes \(Y(t)\) for objects with object and regulator parameters in Table II, with \(K_I = 0.8\) for all cases.

In this case, all the coefficients of the integral link of the regulator were able to “glue” into two fixed values, which simplifies the procedure for the adaptation of the regulator during its implementation and operation.

![Fig. 5. Transient processes \(Y(t)\) for objects with the parameters of the object and the regulator in Table I, despite the fact that for all cases \(K_I = 1\)](image)

![Fig. 6. Transient processes \(Y(t)\) for objects with the parameters of the object and the regulator in Table II, despite the fact that for all cases \(K_I = 0.8\)](image)

**VIII. IDENTIFICATION OF THE BELONGING OF AN OBJECT MODEL TO A GIVEN SUBSET**

Methods and devices for identifying the belonging of the current set of object parameters to a specific subset...
can be based on an estimate of the specified parameters or on another principle, if this is more efficient. In this case, test actions can be used provided they are small enough so as not to disturb the result (accuracy and quality) of control of the object.

For example, the value of the static gain factor $k$ can be approximately determined by applying a small step action to the object. Determining the value of the time constant of the delay link $\tau$ can also be carried out by the response to the step signal, or by the correlation method.

IX. CONCLUSION

The paper proposes a new method of designing adaptive systems, which consists in pre-splitting of the sets of object model parameters into subsets and designing for each subset of robust regulators. At the same time, robust regulators are calculated by numerical optimization when modeling an ensemble of systems of four objects with identical regulators, where the models in these four objects are set from the boundary values of object parameters for the obtained subsets.

REFERENCES


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