Game Theoretic Equilibrium Analysis of Energy Auction in Microgrid

Sanjoy Das, M. Nazif Faqiry, A. Khaled Zarabie, and Hongyu Wu
Electrical & Computer Engineering Dept., Kansas State University, Manhattan, KS 66506, USA
Email: {sdas; mnfaqiry; kzarabie; hongyuwu}@ksu.edu

Abstract—**The future energy grid is expected to be a decentralized network where household units acting as agents can trade energy with others within local neighborhoods by means of an action mechanism. When agents can establish their own price of energy, it is essential to analyze the auction from a market equilibrium standpoint. This paper provides a proof that such a mechanism, although previously formulated as a gradient ascent algorithm to maximize the welfare (i.e. the sum of the utilities of all the agents), converges to the generalized Nash equilibrium (GNE) under physical grid operating constraints, where no agent is incentivized to deviate from its bid. The theoretical analysis is accompanied by simulations of a modified IEEE 37 node system showing convergence towards the equilibrium.**

Index Terms—**generalized Nash equilibrium, online auction, smart grid, projected gradient descent, quasi-variational inequality, multi-agent systems**.

I. Introduction

Due to the increasing penetration of renewable energy resources, the electrical energy distribution system is expected to undergo a transformation from a centralized grid to a decentralized system that operates on market conditions [1]–[3]. Domestic units, i.e. households that utilize energy are equipped with their own renewable energy resources and are sometimes willing to sell surplus generated energy to the grid. Under these circumstances the role of a DSO (distribution system operator) is to ensure that the available energy is distributed in an efficient manner, while the energy grid's physical constraints, such as voltage deviation, transformer capacity and line power limits are within acceptable limits.

A recently proposed market-based approach models the domestic energy consuming units as prosumer agents. These agents can either buy or sell energy through a lower level auction mechanism [4]–[7]. The auction is conducted by an aggregator, which can directly communicate within agents within its physical neighborhood in the grid. The aggregator implements the mechanism as an iterative double auction involving prosumer agents that can act either as buyers or sellers. The auction incorporates asymmetric bidding where some agents receive the unit cost of energy from the aggregator and bid the total amount of energy they wish to trade (power bidders), whereas other agents are assigned by the aggregator, the amount of energy that they can trade, and place as bids unit costs (cost bidders). Other similar algorithmic mechanisms have been proposed recently in smart grid research [8]–[11].

At the upper level, the physical grid incorporates a trading mechanism between the DSO and the aggregators. The overall auction is therefore a bilevel mechanism. The DSO operates in a power setting mode, where it allocates the amount of energy that each aggregator can receive or send to the DSO. The double auction taking place within each aggregator establishes a different unit cost within its own agents, which is returned to the DSO. The entire process is repeated until the upper level algorithm converges to an optimum.

The net energy requirement is submitted by the DSO to the upstream ISO, and separately for each hour for day-ahead energy scheduling. It has been shown in [7] that the overall bilevel mechanism is efficient, i.e. that it maximizes the sum of the utilities of all prosumer agents in the grid.

II. Strategies

A. Prosumer Agents

The prosumer units interact with the grid as selfish agents. The objective of each such agent is to maximize its payoff $\boldsymbol{\pi_k^i}(\cdot)$, which is the difference between the utility gained from consuming energy and the cost of procuring it. The strategy of agent $\boldsymbol{i}$ being served by aggregator $\boldsymbol{k}$ can be cast as the following optimization problem:

$$\text{Maximize: } \pi_k^i(p_k^i) = u_k^i(p_k^i + g_k^i) - c_k p_k^i \tag{1}$$

In the above expression, $u_k^i(\cdot)$ is a utility function that is assumed to be strictly concave and increasing, p_k^i is the amount of energy obtained from the aggregator, and g_k^i is the agent's local energy generation (see Fig. 1). The quantity c_k is the unit cost of energy. Where needed it will be assumed that the local feasibility constraint, $p_k^i + g_k^i = 0$ is not violated. Hence, the prosumer bidding strategy reduces to unconstrained maximization of its payoff as shown above in (1).

Differentiating the expression with respect to energy p_k^i, we have

$$\frac{\partial}{\partial p_k^i} u_k^i(p_k^i + g_k^i) = c_k \tag{2}$$

Manuscript received January 15, 2018; revised March 21, 2018; accepted June 26, 2018.

Corresponding author: Sanjoy Das (email: sdas@ksu.edu).

doi: 10.18178/ijeetc.8.1.39-44

The above expression shows that the optimal bidding strategy is to place bids so that the agent's marginal utility is equal to the unit cost. This is illustrated for buying and selling agents in Fig. 1.

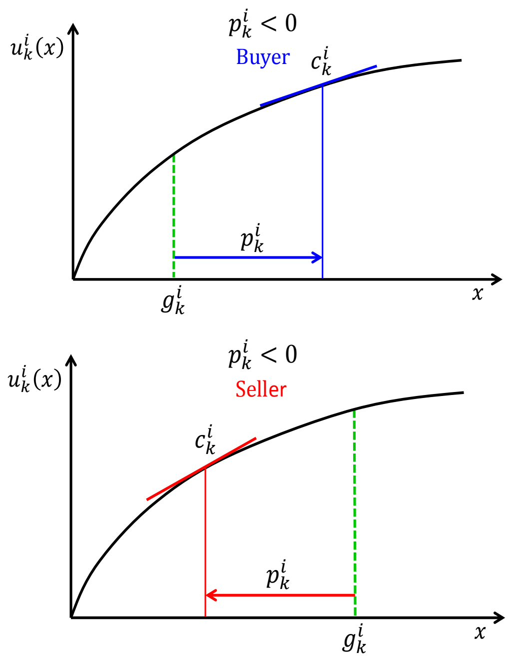

Fig. 1. Agent utilities.

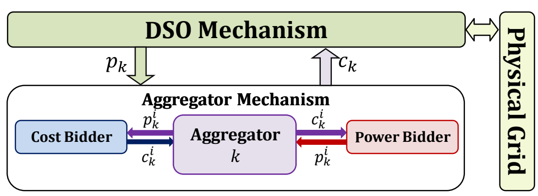

Fig. 2. Schematic of overall trading mechanism.

B. Aggregators

The set of all aggregators in the grid is denoted as $\mathcal{A}$. Each aggregator $k \in \mathcal{A}$ contains a set of prosumer agents $\mathcal{G}_k$. Moreover, $\mathcal{S}_k$ and $\mathcal{D}_k$ are the sets of agents in $\mathcal{G}_k$ that sell and buy energy, so that $\mathcal{S}_k \cup \mathcal{D}_k = \mathcal{G}_k$ and $\mathcal{S}_k \cap \mathcal{D}_k = \emptyset$. It is assumed for simplicity that agents in $\mathcal{S}_k$ are power bidders while those in $\mathcal{D}_k$ are cost bidders. Each aggregator receives the total energy p_k that the DSO allocates to it, and after conducting its own lower level auction, returns the equilibrium cost c_k back to the aggregator (see Fig. 2). The aggregator's strategy is outlined in Algorithm-1 as shown below. In Algorithm-1, $[p_k^i]_{i\in\mathcal{S}_k}$ is the column vector of dimension $|\mathcal{S}_k|$ whose i^{th} entry is p_k^i which is the amount of energy received by the corresponding agent $i \in \mathcal{S}_k$. This convention is followed throughout the remainder of this paper.

The quantity c_k^i is the unit cost of energy reimbursed to each agent $i \in \mathcal{S}_k$. The aggregator sends the unit cost c_k to all agents in $\mathcal{G}_k$ and receives their power bids p_k^i from agents that are willing to sell. The agents are then labeled as buyers or sellers and placed in $\mathcal{D}_k$ or $\mathcal{S}_k$ accordingly.

The total power $p_k - \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}}[p_k^j]_{j\in\mathcal{S}_k}$ that the aggregator is estimated to receive during that iteration, is then allocated to each designated buyer $i \in \mathcal{D}_k$, in proportion to the total monetary amount $p_k^i c_k^i$ that it is willing to spend for procurement. This quantity p_k^i is sent to the buying agents in $\mathcal{D}_k$, which respond by communicating to the aggregator, their unit cost bids, c_k^i. The ratio of the total monetary amount to be procured from the buyers to the energy available from the grid and through the sellers, is the new unit cost c_k.

Algorithm-1 can be viewed as fixed point iteration as shown in Fig. 3. The red curve is the aggregate supply $\mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}}[p_k^j]_{j\in\mathcal{S}_k}$ as a function of the unit cost c_k. It is shifted upwards by an amount p_k, the amount of energy that aggregator k receives from the upper level DSO. The blue curve is the aggregate demand $\mathbf{1}_{|\mathcal{D}_k|}^{\mathrm{T}}[p_k^j]_{j\in\mathcal{D}_k}$.

Algorithm-1: Aggregator Mechanism

Receive from DSO: p_k
Initialize: c_k
Repeat:
    (Obtain supply at cost c_k)
    $\forall i \in \mathcal{G}_k$:
        Send $c_k^i = c_k$
        Receive p_k^i
    (Identify sellers & buyers)
    $\mathcal{S}_k \leftarrow \{i \in \mathcal{G}_k | p_k^i < 0\}$
    $\mathcal{D}_k \leftarrow \{i \in \mathcal{G}_k | p_k^i \text{ not received}\}$
    (Proportionally allocate supply)
    $\forall i \in \mathcal{D}_k$

$$p_k^i \leftarrow \frac{p_k^i c_k^i}{[p_k^j]_{j\in\mathcal{D}_k}^{\mathrm{T}}[c_k^j]_{j\in\mathcal{D}_k}}\left(p_k - \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}}[p_k^j]_{j\in\mathcal{S}_k}\right)$$

    (Obtain cost at demand p_k^i)
    $\forall i \in \mathcal{D}_k$:
        Send p_k^i
        Receive c_k^i
    (Find new cost c_k)

$$c_k \leftarrow \frac{[c_k^i]_{i\in\mathcal{D}_k}^{\mathrm{T}}[p_k^i]_{i\in\mathcal{D}_k}}{p_k - \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}}[p_k^j]_{i\in\mathcal{S}_k}}$$

Until equilibrium
Send to DSO: c_k

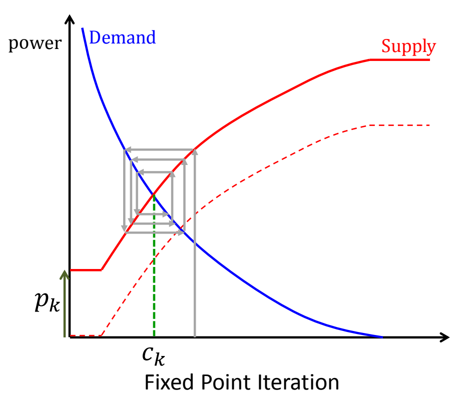

Fig. 3. Aggregator auction as fixed-point iteration.

From Algorithm-1, Clearly the following conditions hold when the auction terminates at the optimum,

$$p_k^i = \frac{p_k^i c_k^i}{\left[p_k^j\right]_{j\in\mathcal{D}_k}^{\mathrm{T}} \left[c_k^j\right]_{j\in\mathcal{D}_k}} \left(p_k - \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}} \left[p_k^j\right]_{j\in\mathcal{S}_k}\right) \quad (3)$$

$$c_k^i = \frac{\left[p_k^j\right]_{j\in\mathcal{D}_k}^{\mathrm{T}} \left[c_k^j\right]_{j\in\mathcal{D}_k}}{p_k - \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}} \left[p_k^j\right]_{i\in\mathcal{S}_k}}. \quad (4)$$

Lemma-1: At the auction's convergence, all agents in $\mathcal{G}_k$, including those in $\mathcal{D}_k$ are charged with the same unit cost c_k.

Proof: From (3),

$$\left[p_k^j\right]_{j\in\mathcal{D}_k}^{\mathrm{T}} [c_k^j]_{j\in\mathcal{D}_k} = c_k^i \left(p_k - \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}} [p_k^j]_{j\in\mathcal{S}_k}\right)$$

$$\Rightarrow c_k^i = \frac{\left[p_k^j\right]_{j\in\mathcal{D}_k}^{\mathrm{T}} [c_k^j]_{j\in\mathcal{D}_k}}{p_k - \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}} [p_k^j]_{j\in\mathcal{S}_k}}.$$

Hence all c_k^i are equal for $i \in \mathcal{D}_k$. From (4), $c_k^i = c_k$.

Lemma-2: Energy balance is satisfied at auction equilibrium.

Proof: This lemma has been established earlier [7] and repeated here for convenience. Replacing all c_k^i with c_k in (4),

$$1 = \frac{1}{\mathbf{1}_{|\mathcal{D}_k|}^{\mathrm{T}} [p_k^j]_{j\in\mathcal{D}_k}} \left(p_k - \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}} [p_k^j]_{j\in\mathcal{S}_k}\right)$$

$$\Rightarrow \mathbf{1}_{|\mathcal{D}_k|}^{\mathrm{T}} [p_k^j]_{j\in\mathcal{D}_k} + \mathbf{1}_{|\mathcal{S}_k|}^{\mathrm{T}} [p_k^j]_{j\in\mathcal{S}_k} = p_k$$

$$\Rightarrow \mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}} \mathbf{p}_k = p_k. \quad (5)$$

Hence, the total power delivered to the agents in $\mathcal{G}_k$ (which is negative for sellers) is equal to the external power received.

The aggregator welfare of each aggregator k is the sum of the utilities of all agents in $\mathcal{G}_k$ so that,

$$\Theta_k(\mathbf{p}_k) = \mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}} \mathbf{u}_k. \quad (6)$$

In the above, $\mathbf{u}_k = [u_k^i]_{i\in\mathcal{G}_k}$. It is established below that the aggregator auction mechanism maximizes the welfare under the constraint that the total energy provided to agents, $\mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}} \mathbf{p}_k$ does not exceed the energy p_k that it receives from the DSO. In other words, Algorithm-1 solves the following constrained optimization problem (COP),

Maximize: $\Theta_k(\mathbf{p}_k) = \mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}} \mathbf{u}_k$.

Subject to: $\mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}} \mathbf{p}_k \le p_k$. (7)

Lemma-3: Given c_k and p_k, the aggregator auction's equilibrium maximizes the welfare $\Theta_k(\mathbf{p}_k)$ [7].

Proof: With ν_k being a dual variable, the Lagrangian of the COP in (6) is given by,

$$\mathcal{L}_k(\mathbf{p}_k, \nu_k) = \mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}} \mathbf{u}_k - \nu_k\left(\mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}} \mathbf{p}_k - p_k\right) \quad (8)$$

The stationary condition of (7) for each agent $i \in \mathcal{G}_k$ is,

$$\frac{\partial u_k^i}{\partial p_k^i} = \nu_k. \quad (9)$$

Since $\Theta_k(\cdot)$ is the sum of strictly concave functions, the COP in (6) has a unique maximum, which satisfies (9). Comparing (4) with (9), establishes Lemma-3 with $\nu_k = c_k$.

C. DSO

The DSO receives the vector $\mathbf{c} = [c_k]_{k\in\mathcal{A}}$ of unit costs from the aggregators and provides the energy amounts specified by the vector $\mathbf{p} = [p_k]_{k\in\mathcal{A}}$. In doing so, it must not violate the physical grid constraints. Suppose the grid constraints are represented as the vector of inequalities $\mathbf{h}(\mathbf{p}) \le \mathbf{0}$, Algorithm-2 below shows the various steps of the DSO mechanism.

Algorithm-2: DSO Mechanism

```
Repeat:
    Receive from aggregators: c.
        γ ← c
    Gradient step:
        p ← P_F(p + ηγ)
    where,
        F = {p | h(p) ≤ 0,   c^T p ≤ c_0 1^T_{|A|} p}
    Send to aggregators: p.
Until converged
```

The operator $P_{\mathcal{F}}(\cdot)$ is the projection of its argument to the nearest point in the feasible region $\mathcal{F}$.

Consider the following COP,

Maximize: $\mathbf{1}_{|\mathcal{A}|}^{\mathrm{T}} \mathbf{\Theta}(\mathbf{p})$.

Subject to: $\mathbf{h}(\mathbf{p}) \le \mathbf{0}$,

$\mathbf{c}^{\mathrm{T}}\mathbf{p} \ge c_0 \mathbf{1}_{|\mathcal{A}|}^{\mathrm{T}}$. (10)

The following lemma illustrates how Algorithm-2 implements gradient descent towards the optimal value of $\mathbf{p}$.

Lemma-4: The total welfare is maximized at the bilevel mechanism's optimum.

Proof: From (6) it is seen that,

$$\nabla_{\mathbf{p}} \mathbf{\Theta}(\mathbf{p}) = \left[\frac{\partial \Theta_k}{\partial p_k}\right]_{k\in\mathcal{A}} = [\nu_k]_{k\in\mathcal{A}}.$$

But using (3) and (9),

$$\mathbf{c} = [\nu_k]_{k\in\mathcal{A}}.$$

HENCE,

$$\nabla_{\mathbf{p}} \mathbf{\Theta}(\mathbf{p}) = \mathbf{c}. \quad (11)$$

The vector of unit costs that Algorithm-1 converges to provides the gradient direction for Algorithm-2. The DSO MECHANISM in Algorithm-2 is a straightforward implementation of projected gradient ascent where $\mathbf{c}$ is the gradient direction [12], [13].

III. GAME THEORETIC FORMULATION

A. Generalized Nash Equilibrium

Consider a game $(\mathcal{A}, \mathcal{X}_i, u_i)$, where $\mathcal{A}$ is a set of selfish agents, and for every $i \in \mathcal{A}$, its action x_i is in the set $\mathcal{X}_i$ and $u_i: \mathcal{X}_i \to \Re$ is its payoff function. Generalized Nash equilibrium (GNE) extends this concept by constraining the set of actions available to each agent i to be dependent on the actions x_{-i} of all other agents [14]. Hence, we may write, $x_i \in \mathcal{X}_i(x_{-i})$.

With $\mathbf{x} \triangleq [x_i]_{i\in\mathcal{A}}$ and $\nabla\mathbf{u} \triangleq [\nabla u_i]_{i\in\mathcal{A}}$, the GNE conditions can be expressed in the following manner.

$$\forall i \in \mathcal{A}, \forall x_i \in \mathcal{X}_i(x_{-i}), \nabla u_i(x_i^*)(x_i - x_i^*) \geq 0.$$

The above GNE condition can be shown to reduce to the following quasi-variational inequality $QVI(\mathcal{X}_i, \nabla\mathbf{u})$ [16] (also see Fig. 4),

$$\forall \mathbf{x} \in \prod_i \mathcal{X}_i(x_{-i}), \qquad \nabla\mathbf{u}^{\mathrm{T}}(\mathbf{x}^*)(\mathbf{x}^* - \mathbf{x}) \geq 0. \tag{12}$$

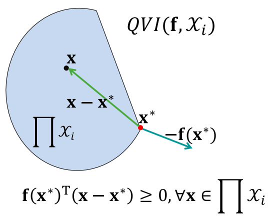

Fig. 4. Quasi-variational inequality problem.

B. *Aggregator Equilibrium*

A game $\mathbb{G}_k(\mathcal{G}_k, \mathcal{P}_k^i, u_k^i)$ can be defined at each aggregator, where using (7) the set of feasible actions is given be the following expression.

$$\mathcal{P}_k^i = \{p_k^i | \mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}} \mathbf{p}_k \leq p_k\}. \tag{13}$$

The GNE of $\mathbb{G}_k$ at each aggregator is shown in the following theorem.

Theorem-1: The aggregator mechanism in Algorithm-1 establishes GNE.

Proof: Let $\mathbf{p}_k^*$ be the equilibrium energy consumptions of the agents and $\mathbf{q}_k \in \prod_i \mathcal{P}_k^i$ another feasible vector of consumptions. From (2),

$$\nabla_{\mathbf{p}_k}\mathbf{u}_k = c_k \mathbf{1}_{|\mathcal{G}_k|}$$

$$\Rightarrow \nabla_{\mathbf{p}_k}\mathbf{u}_k^{\mathrm{T}}(\mathbf{p}_k^* - \mathbf{q}_k) = c_k \mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}}(\mathbf{p}_k^* - \mathbf{q}_k).$$

Using (5), the above is equivalent to the expression below.

$$\nabla_{\mathbf{p}_k}\mathbf{u}_k^{\mathrm{T}}(\mathbf{p}_k^* - \mathbf{q}_k) = c_k p_k - c_k \mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}}\mathbf{q}_k.$$

Since $\mathbf{q}_k$ is feasible, $\mathbf{1}_{|\mathcal{G}_k|}^{\mathrm{T}}\mathbf{q}_k \leq p_k$, whence from the above,

$$\nabla_{\mathbf{p}_k}\mathbf{u}_k^{\mathrm{T}}(\mathbf{p}_k^* - \mathbf{q}_k) \geq 0. \tag{14}$$

Upon comparing the above expression with (12), clearly Algorithm-1 is a solution to $QVI(\mathcal{P}_k^i, \nabla\mathbf{u}_k)$. This proves that Algorithm-1 establishes GNE.

C. *DSO Equilibrium*

Consider the game $\mathbb{G}_{\mathrm{DSO}}(\mathcal{A}, \mathcal{F}, \Theta_k)$. It will now be established that $\mathbf{p}^*$ is the GNE solution by means of the following theorem.

Theorem-2: The DSO mechanism in Algorithm-2 establishes GNE.

Proof: Since it has been shown that Algorithm-1 maximizes Θ_k using its allocated power p_k, we consider it to be a function of the latter and indicate this as $\Theta_k(p_k)$.

Let the following expression be the solution of the constrained optimization problem in (10) that is arrived at by Algorithm-2.

$$\mathbf{p}^* = \underset{\mathbf{p}\in\mathcal{F}}{\operatorname{argmax}}\, \mathbf{1}_{|\mathcal{A}|}^{\mathrm{T}}\boldsymbol{\Theta}(\mathbf{p}), \tag{15}$$

Next, let $\mathbf{q} \in \mathcal{F}$ be another feasible allocation of power to the aggregators. Since $\mathbf{p}^*$ solves the constrained optimization problem in (10), according to the minimum principle,

$$\nabla_{\mathbf{p}}\boldsymbol{\Theta}(\mathbf{p}^*)^{\mathrm{T}}(\mathbf{p}^* - \mathbf{q}) \geq 0 \tag{16}$$

Hence $\mathbf{p}^*$ is a solution to $QVI(\mathcal{F}, \nabla\boldsymbol{\Theta})$. In other words, it is at GNE.

Other approaches to obtain GNE are provided in [15], which may be considered as an alternative approach where the feasibility constraints are included as an augmented penalty term in the Lagrangian.

IV. Results & Conclusion

A modified IEEE 37 node system has been used to simulate the proposed bilevel mechanism [7] (see Fig. 5). The system contains 17 aggregators.

A set of agents were randomly generated for each aggregator. A total of 483 agents with 303 buyers and 180 sellers, each with its own utility curve and generation was generated. Four scenarios, labeled I, II, III, and IV were created with increasing price levels [7]. The result of the application of Algorithm-2 with each aggregator executing a local copy of Algorithm-1 is provided in Fig. 6. The convergence towards GNE is clearly observed.

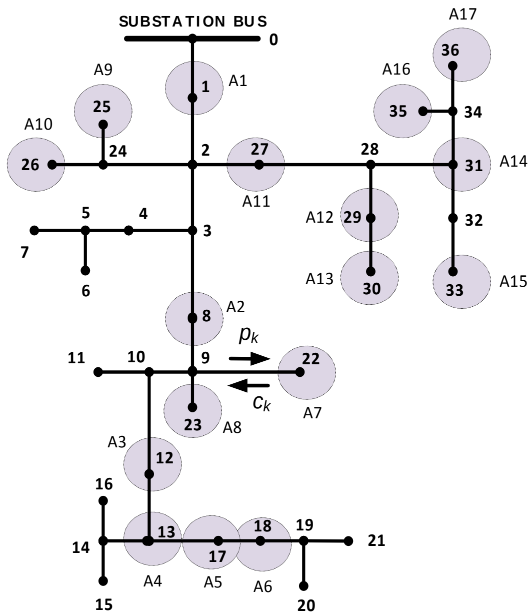

Fig. 5. Modified IEEE 37 node system.

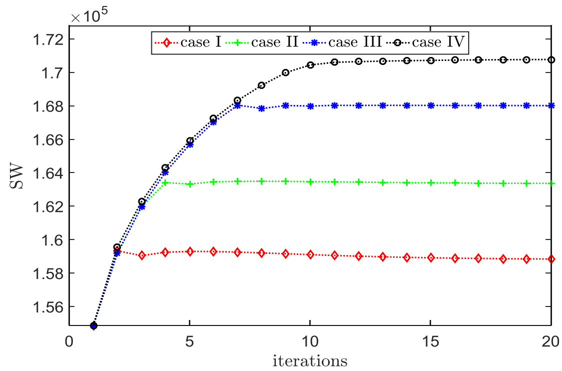

Fig. 6. Convergence towards GNE for four cases.

Prior research work in [7] has already shown the effectiveness of the auction mechanism discussed here. However, it was not specifically shown that the approach converges towards the GNE. In proving this result, this research shows that the algorithm converges towards the efficient optimum that is also stable from a market standpoint, where no agent would arbitrarily change its declared bid.

ACKNOWLEDGEMENT

This work was partially supported by the National Science Foundation-CPS under Grant CNS-1544705.

REFERENCES

[1] A. Pahwa, S. A. Deloach, B. Natarajan, S. Das, A. R. Malekpour, S. M. S. Alam, and D. M. Case, "Goal-Based holonic multiagent system for operation of power distribution systems," *IEEE Trans. on Smart Grid*, vol. 6, no. 5, pp. 2510–2518, 2015.

[2] W. Saad, Z. Han, H. V. Poor, and T. Basar, "Game-Theoretic methods for the smart grid: An overview of microgrid systems, demand-side management, and smart grid communications," *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 86–105, 2012.

[3] X. Fang, S. Misra, G. Xue, and D. Yang, "Smart grid—The new and improved power grid: A survey," *IEEE Communications Surveys & Tutorials*, vol. 14, no. 4, pp. 944–980, 2012.

[4] M. N. Faqiry and S. Das, "Double-Sided energy auction in microgrid: Equilibrium under price anticipation," *IEEE Access*, vol. 4, pp. 3794–3805, 2016.

[5] M. N. Faqiry and S. Das, "A budget balanced energy distribution mechanism among consumers and prosumers in microgrid," in *Proc. IEEE Int. Conf. on Cyber, Physical and Social Computing* (CPSCom), Chengdu, China, 2016, pp. 516–520.

[6] M. N. Faqiry, A. K. Zarabie, F. Nassery, H. Wu, and S. Das, "A day ahead market energy auction for distribution system operation," in *Proc. IEEE Conf. on Electro-Information Technology*, Lincoln, USA, 2017, pp. 14–17.

[7] M. N. Faqiry and S. Das, "Distributed bilevel energy allocation mechanism with grid constraints and hidden user information," *IEEE Trans. Smart Grid*, 2017, in press.

[8] M. N. Faqiry and S. Das, "Double auction with hidden user information: Application to energy transaction in microgrid," *IEEE Trans. Systems, Man & Cybernetics, Systems*, 2018, in press.

[9] K. Boroojeni, M. Hadi Amini, A. Nejadpak, T. Dragičević, S. S. Iyengar, and F. Blaabjerg, "A novel cloud-based platform for implementation of oblivious power routing for clusters of microgrids," *IEEE Access*, vol. 5, pp. 607–619, 2017.

[10] M. N. Faqiry, L. Edmonds, H. Zhang, A. Khodaei, and H. Wu, "Transactive market based operation of distributed electrical energy storage with grid constraints," *Energies*, vol. 10, no. 11, pp. 1891, 2017.

[11] S. Zou, Suli, Z. Ma, and X. Liu, "Resource allocation game under double-sided auction mechanism: efficiency and convergence," *IEEE Trans. Automatic Control*, vol. 63, no. 5, pp. 1273–1287, 2018.

[12] D. P. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE J. Selected Areas Communications*, vol. 24, no. 8, pp. 1439–1451, 2006.

[13] S. Singh, M. Kearns, and Y. Mansour, "Nash convergence of gradient dynamics in general-sum games," in *Proc. 16th Conf. on Uncertainty in Artificial Intelligence*, Morgan Kaufmann Publishers Inc., 2000, pp. 541–548.

[14] F. Facchinei and C. Kanzow. "Generalized nash equilibrium problems," *Annals of Operations Research*, vol. 175, no. 1, pp. 177–211, 2010.

[15] C. Kanzow and D. Steck, "Augmented lagrangian methods for the solution of generalized nash equilibrium problems," *SIAM Journal on Optimization*, vol. 26, no. 4, pp. 2034–2058, 2016.

[16] G. Scutari, D. P. Palomar, F. Facchinei, and J. Pang, "Convex optimization, game theory, and variational inequality theory," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 35-49, 2010.

Sanjoy Das received his Ph.D. in Electrical and Computer Engineering from Louisiana State University, Baton Rouge, USA in 1994. Between 1994 and 1997, he received post-doctoral training at the University of California, Berkeley, USA, where he focused on neural networks and neuro-motor control.
After serving in the industry as a research scientist, he joined the Electrical and Computer Engineering Department at Kansas State University in 2001, where he is currently an Associate Professor. His research interests include machine learning, mathematical aspects of data science, neural networks, multi-agent systems, algorithmic game theory and quantum computing.
Dr. Das has published over 60 journal papers and 100 conference papers. He has served in the technical committees and editorial boards in various international conferences and technical journals.

M. Nazif Faqiry received his B.S. in Electrical & Electronics Engineering from Kabul University, Afghanistan, in 2007, and his M.S. and Ph.D. degrees in Electrical & Computer Engineering from Kansas State University in 2010 and 2017. Between 2011 and 2014, he served as a lecturer at Kabul University, where he taught several senior level courses in electrical engineering. He also worked as a senior electrical engineer at State Corps Engineering, Afghanistan, between 2011 and 2013. From 2017 to 2018, Faqiry worked as a Post-doctoral researcher at Kansas State University.
He is currently a Research Assistant Professor at the Electrical and Computer Engineering Department at Kansas State University.
Dr. Faqiry's research interests are in the applications of game theory & mechanism design, network economics, and machine learning to power market operations and smart grid.

A. Khaled Zarabie received his B.S. in Electrical & Electronics Engineering from Kabul University, Afghanistan, in 2007, and his M.S. degree in Electrical & Computer Engineering from Michigan Technological University in 2012. He worked as instructor at Engineering Faculty, Kabul University, where he taught courses in electrical engineering between 2013 and 2016. He also worked as a senior electrical engineer at ZNCC/State Corps Engineering, Afghanistan, for 2 years.
Since 2016, he is working towards his Ph.D. degree in Electrical & Computer Engineering at Kansas State University.
Dr. Zarabie's research interests are in game theory and mechanism design, machine learning, active consumer modelling and transactive energy market operation.

Hongyu Wu received the B.S. degree in energy and power engineering and the Ph.D. degree in systems engineering, both from Xi'an Jiaotong University, Xi'an, China, in 2003 and 2011, respectively.
He is currently an Assistant Professor in the Department of Electrical and Computer Engineering, Kansas State University (K-State), Manhattan, KS, USA. From 2011 to 2014, he was a Postdoctoral Researcher in the Robert W. Galvin Center for Electricity Innovation, Illinois Institute of Technology in Chicago, Chicago, IL, USA. Before

joining K-State, he worked as a Research Engineer in the Power Systems Engineering Center, National Renewable Energy Laboratory. Dr. Wu's research interests include modeling and optimization of large-scale systems, power system operation and control, home energy management, and smart grid technologies.