Abstract—In this article, a model simulating wave propagation in an infinite isotropic structure excited by a thin piezoelectric wafer is considered. Three different models are used to calculate wave fields occurring in the considered layer. The first finite element (FE) model simulates the excitation caused by a piezoelectric actuator bonded onto the host structure using an adhesive layer. The actuator is driven by the electric potential applied to its upper surface. The second semi-analytical model is based on the Fourier transform and on the Green’s matrix representation for the calculation of the occurring displacement fields. The third model introduces the effect of the actuator as two pin forces applied to the ends of the actuator. The results demonstrate the advantages and disadvantages of the presented approaches, as well as the effect of a bonding layer in a wide frequency range.

Index Terms—composite structure, finite element modeling, Fourier transform, Green’s matrix, perfectly matched layer (PML), piezoelectric actuator, pin-force model, waves excitation

I. INTRODUCTION

Condition Monitoring methods (CM) comprise the techniques allowing to monitor, detect and analyze machinery condition data [1], [2]. Application of condition monitoring helps to avoid losses caused by the breakdown of industrial machinery and to reduce maintenance costs. Nowadays, the use of travelling waves is one of the most widespread methods for detecting damages in engineering systems [3]-[5]. Among the large number of existing methods for elastic waves excitation, the method based on the use of piezoelectric elements remains the most frequently used one [4]-[10] because these transducers have compact size, high sensitivity over the wide frequency range and low production costs. When used as actuators, piezoelectric elements are adhesively bonded on the inspected structure to convert the driving electric voltage into mechanical strain. The excited waves go through the structure; if they meet a damage, such as delamination, void or inclusion, they interact with it. The subsequent extraction of damage information is performed by comparing the delay of the arrival time of signals before and after damage and the wave attenuation. Numerous investigations are devoted to the interaction between a piezoelectric sensor, or an actuator, and an elastic structure. They prove that shear stresses occurring between a piezoelectric patch and a host structure concentrate at the edge of the contact area [9]. Thereby, the simplified so called pin-force model [5], [11] is widely used to excite different kinds of acoustic waves in elastic structures. In this model, the deformation is transferred from a piezoelectric patch to the host structure through the forces “pins” concentrated at the ends of the actuator. However, only in a few works [12]-[14] the effect of a bonding layer between a piezoelectric patch and the structure is taken into account. The purpose of this paper is to estimate the effect of the glue and to analyze wave fields occurring in an isotropic infinite layer using three different approaches.

In the first approach, the considered isotropic structure is excited by a thin piezoelectric patch mounted on the upper surface. In this model, for two infinitely far ends of the layer, an absorbing boundary condition is applied to simulate open boundaries, and is described by means of perfectly matched layers (PML). The wave field is obtained by means of the steady-state analysis performed in the FE package COMSOL Multiphysics. Different values of the glue thickness are taken into consideration. The second semi-analytical method makes it possible to calculate unknown displacement fields based on the Fourier transform, the Green’s matrix, and the numerical contour integration. The third approach is the simplified approach, which describes the effect of a thin actuator using the pin-force model. The occurring wave fields are analyzed at different vibration frequencies.

II. PROBLEM FORMULATION

An isotropic infinite layer of thickness $h$, which occupies the volume $D = \{x, z\mid -\infty < x < \infty; -h \leq z \leq 0\}$ is considered. The oscillations of the layer are excited by a thin piezoelectric actuator of thickness $h_{PZT}$ mounted on the upper surface in the region $[-a, a]$ . The glue thickness is $h_{gs}$. Time dependency is assumed harmonic in the form $e^{-j\omega t}$, where $\omega$ is the vibration frequency, as shown in Fig. 1.
Let us consider one half of an isotropic layer of thickness $h=2$ mm and length $l=1$ m actuated by a piezoelectric wafer. Elastic properties of the layer are taken as follows: mass density $\rho=2500$ kg/m$^3$, Poisson’s ratio $\nu=0.33$ and Young’s modulus $E=20$ GPa.

Below the analysis of the wave fields numerically calculated for the three considered models is presented in detail.

A. Comparison of the Results Obtained for the FE-Model with and without an Adhesive Layer

According to the first FE model, an unknown wave-field is obtained by the frequency response analysis. This approach is used for both models - with and without a bonding layer - and for different glue thicknesses.

In the first model the actuating patch of a half-length $a=10$ mm and thickness $h_{PZT}=0.2$ mm is made of piezoelectric ceramics PZT-5H, when the properties of a bonding layer are: mass density $\rho=910$ kg/m$^3$, Poisson’s ratio $\nu=0.37$ and Young’s modulus $E=1.02$ GPa. The thickness of the bonding layer varies in the range: $h_1=0$ mm that corresponds to a model without adhesive layer, $h_2=10$ mm and $h_3=50$ mm. On the top of the PZT-actuator, an electric potential with the amplitude $U=50$ V is applied when the bottom surface is grounded. The piezoelectric constant of PZT-5H used in the patch is $d_{33}=265$ mm/kV. A perfectly matched layer (PML) of a finite length $l=0.5$ m simulates a reflectionless boundary condition on the right side of the layer. A symmetry plane boundary condition is applied to the left side of the structure. All other boundaries are assumed to be free. In Fig. 2 the mapped finite-element mesh of a part of the considered structure is presented. Minimum number of finite elements in vertical direction for the bonding layer is taken equal to 3 for the finite element model with $h_3=50$ mm. Influence of the number of finite elements on the final solutions is estimated. Below in Table I different sizes of the finite element mesh used during the FE-analysis are presented. In case of the extremely coarse mesh the lowest number of quadrilateral elements is taken. This case corresponds to the lowest number of degrees of freedom and the longest calculation time. The case of extremely fine mesh corresponds to the highest number of the finite elements, number of degrees of freedom and the shortest calculation time.

![Figure 1. Scheme of the loaded structure.](image1)

![Figure 2. Part of the mapped FE-mesh near the contact area.](image2)

<table>
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<tr>
<th>No</th>
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<th>Number of degrees of freedom</th>
<th>Number of elements</th>
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</tr>
<tr>
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<td>10394</td>
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<tr>
<td>6</td>
<td>Extra fine</td>
<td>184935</td>
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<tr>
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</table>

In Fig. 3 dependencies of the $x$-displacements calculated on the upper surface of the host structure are presented depending on the finite element mesh size. The numbers presented in the legend correspond to the...
Displacement fields are calculated at various vibration frequencies: (a) \( f = 35 \) kHz, (b) \( f = 75 \) kHz, and (c) \( f = 150 \) kHz. From Fig. 3 (a) one can see that for the frequency \( f = 35 \) kHz all the used meshes give very similar distributions of the displacement fields. It means that even a coarse mesh may lead to the reasonable results at low frequencies. For the higher frequency \( f = 75 \) kHz the results corresponding to the coarse and normal size mesh differ significantly and they are stabilized when the fine mesh is used (see Fig. 3 (b)). In case of the highest considered vibration frequency \( f = 150 \) kHz already an extra fine FE-mesh is needed (see Fig. 3 (c)). This is due to the fact that the size of the largest element has to be substantially smaller than the wavelength in order to resolve the problem. For the subsequent analysis the most suitable FE-mesh density is used at any particular frequency. Relatively small computational time costs needed for modeling the oscillations of the isotropic layer are rapidly increasing in case of multilayer composites.

Below in Fig. 4, shear stresses in the contact area are presented depending on the glue thickness at two different frequencies (a) \( f = 5 \) kHz and (b) \( f = 75 \) kHz. It is apparent that a thicker bonding layer produces a weakened load transfer between the patch and the host structure. In the models without bonding and with a thinner adhesive layer of thickness \( h_b = 10 \) µm, shear stresses are concentrated at the end points of a piezoelectric patch.
amplitude of $x$-displacement takes the highest value, whereas the thickest bonding layer with $h_b=50 \, \mu m$ sufficiently attenuates the tangent displacements of the lower actuator's surface, and reduces the surface displacements of the host structure. In case of a higher vibration frequency the contact displacements take a more complicated shape and thus can no longer be described by a linear function. During the research, graphs for the $y$-components of contact stresses and displacements were plotted. Their amplitudes are much smaller (approximately ten times) than the amplitudes of the $x$-components. Therefore, when constructing a semi-analytical model and two pin forces, the $y$-components are not taken into account.

Surface plots of $x$- and $y$-displacements of the host structure are presented in Fig. 6. The FE-analysis is performed at vibration frequencies (a) $f=35 \, kHz$ and (b) $f=75 \, kHz$ in case of a bonding layer of $h_b=10 \, \mu m$.

The deformed shapes of the considered layer are presented in a rainbow color table. The initial undeformed boundaries are represented as the black rectangle. Under each surface plot the scale of amplitudes in meters is presented. It is obvious that the amplitudes of the host structure surface $x$-displacements are considerably higher than the $y$-displacements. In Fig. 7 displacement fields calculated over the upper surface of the host structure at vibration frequencies (a) $f=35 \, kHz$, (b) $f=75 \, kHz$ and (c) $f=150 \, kHz$ with varied thickness of the adhesive layer are presented. The graphics indicate that the bond thickness has not much influence on the resulting displacements in far field at lower vibration frequencies (a). With larger frequency (b) the discrepancy gets higher. For the model without a glue (b) the highest amplitudes of vibrations are presented, when
for the models with thicker bonding layers of \( h_b = 10 \, \mu m \) and \( h_b = 50 \, \mu m \) the resulting displacement fields almost coincide and look smoother around the peaks (b). The strongest discrepancy (c) in smoothness of the resulting displacement fields corresponds to the highest vibration frequency, when the maximal amplitudes take very close values for all considered models. It is obvious that the thicker adhesive layer leads to a slightly smoother displacement field.

B. Comparison of the Numerical Results: FE-Model, Integral Representation and Pin-Force Model

In the second approach, the formulated problem (1)-(5) is solved using a semi-analytical approach based on the use of the Fourier transform [7], [15] applied to (1) with respect to the spatial coordinate \( x \) and the Fourier variable \( \alpha \). The solution of the problem can be written as follows

\[
u(x, z) = \frac{1}{2\pi i} \int_{\Gamma} K(\alpha, z) Q(\alpha) e^{-i\alpha x} d\alpha
\]

where \( K \) and \( Q \) are the Fourier transforms of the Green’s matrix \( k \) and of the function of load \( q(x) \), respectively.

\[
Q = \int_{-\alpha}^{\alpha} q(x)e^{i\alpha x} dx = \begin{bmatrix} Q_0 & 0 \end{bmatrix}^T,
\]

\[
K = K_{11} = -\frac{iM_1(\alpha, z)}{\Delta(\alpha)}
\]

where

\[
M_1(\alpha, z) = -i\sigma_x \left\{ \left( \alpha^2 - \kappa_1^2 \right) \sinh(\sigma_z) + \gamma^4 \sinh(\sigma_z) \right\} - \alpha^2 \gamma^4 \cosh(\sigma_z) \sinh(\sigma_0(z + h)) + \alpha^2 \sigma_z \sinh(\sigma_z) \cosh(\sigma_0(z + h)) - \alpha^2 \gamma^4 \cosh(\sigma_z) \sinh(\sigma_0(z + h)) + \gamma^4 \cosh(\sigma_z) \sinh(\sigma_0(z + h)) \right\},
\]

\[
\Delta(\alpha) = 2\mu \left\{ -2\alpha^2 \sigma_z \gamma^4 - \gamma^4 - \alpha^4 \sigma_x^2 \right\} \sinh(\sigma_0) \cosh(\sigma_0) + 2\alpha^2 \sigma_z \gamma^4 \cosh(\sigma_0) \sinh(\sigma_0),
\]

where \( \gamma^2 = \alpha^2 - \omega^2 / \mu \), \( \sigma_x^2 = \alpha^2 - \kappa_1^2 \), \( \sigma_z^2 = \alpha^2 - \kappa_2^2 \), \( \kappa_1 = \omega / v_p \) and \( \kappa_2 = \omega / v_s \) are the wavenumbers for the longitudinal and shear waves respectively, \( v_p \) and \( v_s \) are the velocities of their propagation, \( \mu \) is the shear modulus, \( \omega \) is the dimensionless vibration frequency. The load \( q(x) \) is taken from the above described FE-problem and approximated by a system of the basis functions. In accordance with the limiting absorption principle [16], the integration contour \( \Gamma \) goes in the complex plane \( \alpha \) along the real axis. It bypasses the positive poles of \( K \) from below, and the negative ones - from above in case without backward waves (see Fig. 8).

In the third model, the contact stresses appearing under the actuating patch are described by means of a simplified approach, according to which the bonding between a thin piezoelectric patch and a waveguide is assumed to be ideal and the shear stress distribution along the surface of an actuator can be expressed using two Dirac delta-functions

\[
r_{-\alpha} |_{\chi=0}(x) = a \tau_0 \left[ \delta(x-a) - \delta(x+a) \right]
\]

where \( a \tau_0 \) is the pin force applied at the boundary points \( x = \pm \alpha \) of the contact area [5], [9], [11]. The pin force amplitudes are calculated by the integration of shear stresses appearing in the contact area. These stresses are taken from the aforementioned FE-model. The numerically calculated wave fields are compared for the three considered models at vibration frequency \( f = 35 kHz \). The results corresponding to two different values of the glue thickness are presented in Fig. 9: (a) \( h_b = 0 \, \mu m \) and (b) \( h_b = 50 \, \mu m \).

One can see from Fig. 9 (a) that the amplitudes of displacements are in a good agreement for all the three considered models when the thickness of bonding layer
equals to zero. In Fig. 9 (b) one can see that results obtained by integral approach and FE-model are in a good agreement both in far and near field, when the results calculated using a pin-force model differ significantly in the vicinity of a piezoelectric patch. It is obvious that all three approaches allow to obtain comparable results in a far field.

IV. CONCLUSIONS

In this work, the effect of a bonding layer between an infinite isotropic structure and a piezoelectric thin patch actuator is investigated. The results indicate that the thickness of the bonding layer has a significant effect on actuation characteristics of the piezoelectric patch.

The shear stress distribution under the piezoelectric actuator differs considerably with the growth of the thickness of a bonding layer. It is shown that the adhesive layer of the highest thickness produces the dumped load transfer between the patch and the structure. When the thickness of the bonding layer is equal to zero, shear stresses concentrate in the vicinities of the end points of the contact area and the load transfers to the surface of the host structure predominantly over these points.

An influence of the finite element mesh density on the distribution of the displacement fields is analyzed. The element size limits are estimated for every considered frequency range.

The resulting displacements in the far field are not much influenced by the thickness of the bonding layer, whereas near the vibration source the difference of the waves behaviour is obvious. It is stated that the thicker adhesive layers allow to obtain the slightly smoother displacement fields, and an influence of adhesive thickness on the wave amplitudes increases with the growth of vibration frequency.

Three different approaches are used to simulate the excitation of the host structure. The resulting displacement fields are compared for all simulated models at one vibration frequency and with varied thickness of a bonding layer.

Analysis of the obtained results shows that all the applied methods can be effectively used when the thickness of an adhesive layer is negligible, and a so-called ideal bonding takes place. It is shown that with the growth of the bond thickness the FE-model and the integration approach lead to the comparable results both near the vibration source and in a far field, when the pin-force model leads to the high discrepancy in the near field.

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REFERENCES


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