Delay-Dependent Stability and Performance Analysis for Time Delay Systems Interconnected over an Undirected Graph

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***Abstract*—Time delay is usually inevitable in a system consisting of spatially and/or wirelessly interconnected units or subsystems. This paper deals with the fundamental problem of stability and contractive performance for systems interconnected over an undirected graph with time delay in the states of subsystems. A sufficient condition for the well-posedness, delay-dependent stability and contractiveness of such systems is derived by means of finite-dimensional linear matrix inequalities (LMIs), which provides a useful tool for further exploration of the controller design problem. A numerical example is also presented to show the validity of the obtained results.**

***Index Terms*—delay-dependent stability, contractiveness, time delay systems, undirected graph, LMIs**

I. Introduction

The recent years have witnessed a great deal of activities in the study of spatially and/or wirelessly interconnected systems, such as satellites in formation [1], unmanned aerial vehicles in formation [2], robots in formation [3], and automated highways [4]. Such interconnected systems present usually complex global dynamic behavior since the subsystems included therein have the characteristic of sensing, computing and communicating, etc. And these systems can be described by means of an undirected graph, the state-space representations of the different subsystems and an interconnection condition (see, e.g., [5]), which is often simply called systems interconnected over graphs.

With respect to these systems interconnected over graphs, a lot of research results have been obtained for the problems of stability, performance analysis and controllers design. For examples, the problems of synthesizing a distributed output feedback controller achieving $H\infty$ performance have been considered for continue and discrete time system interconnected over arbitrary graph structures in [5] and [6], respectively. The problem of analysis, synthesis and implementation of distributed controllers for homogeneous and heterogeneous interconnected systems with a highly structured interconnection topology have been dealt with in [7] and [8], respectively.

Time delay, which generally gives rise to deteriorating the system performance, is very common in practical dynamical systems. Some research topics on time delay have been investigated (see, e.g., [9]–[12]). Regard to the interconnected systems, time delay may appear in the state variables of the subsystems as well as in the inter-connections. The problem of distributed controllers design for these systems with arbitrarily small communication delays between subsystems has been considered in [13], [14]. However, with respect to the system inter-connected over an undirected graph, few results are known for the case where time delay appears in the state variables of the subsystems.

In this paper, we consider a system consisting of L different or similar subsystems with their own time delay, interconnected over an undirected graph, and the main purpose is to find a sufficient condition for the system to achieve the well-posedness, delay-dependent stability and a desire performance. The paper is organized as follows. In Section II, some preliminaries and definitions are presented. In Section III, a sufficient condition is given in terms of LMIs for the well-posedness, delay-dependent stability and contractiveness of the time delay systems interconnected over an undirected graph. In Section IV, a numerical simulation is provided to illustrate the effectiveness of the proposed result. Finally, the conclusion is given in Section V.

Notation: $\mathbb{R}$ denotes the set of real numbers. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The non-negative real number is denoted by $\mathbb{R}^+$ and the $n\times m$ real matrix is denoted by $\mathbb{R}^{n\times m}$. The $n\times n$ positive definite matrix is denoted by $\mathbb{R}_+^{n\times n}$. I and 0 stand for the identity and zero matrix, respectively. The set of $n\times n$ real symmetric matrices is denoted by $\mathbb{R}_S^{n\times n}$. Given real symmetric matrix $H, H>0$ means $x^* Hx > 0$ for all $x\neq 0$. When $Y_i, i=1,2,\cdots,L$ are given, $\mathrm{diag}_{k<i<l} Y_i$ is defined by block-diagonal matrix. Likewise, $\mathrm{col}_{k<i<l} u_i$ denotes the signal $(u_k,\cdots,u_l)$ formed by concatenating u_i.

$\|\bullet\|$ denotes the Euclidean norm. The signals dealt with in this paper belong to the class $\mathcal{L}_2$, where n dimension-

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al functions $x(t)$ mapping $\mathbb{R}^+$ to $\mathbb{R}^n$ for which the following quantity is satisfied: $\int_0^\infty \| x(t)\|^2 dt < \infty$, and the inner product of $x(t)$ and $y(t)$ on $\mathcal{L}_2$ is defined as $\langle x(t), y(t)\rangle_2 := \int_0^\infty \langle x(t), y(t)\rangle dt$ with corresponding norm $\| x(t)\|_2 := \sqrt{\langle x(t), x(t)\rangle_2}$.

II. Preliminaries

The time delay system Σ concerned here is composed of an assembly of L different or similar linear time invariant (LTI) subsystems with their own time delay. The subsystems are interconnected over an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{N})$. The set $\mathcal{V}$ of vertices is defined with the element $\{\Sigma_i, i = 1, 2, \cdots, L\}$, where each Σ_i is a LTI finite dimensional subsystem. The set $\mathcal{E}$ of non-oriented edges is defined as $\{(\Sigma_i, \Sigma_j), i \le j\}$. Each edge (Σ_i, Σ_j) is weighted by n_{ij}, which denotes the dimension of the output of Σ_i flowing towards Σ_j, and $n_{ij} = n_{ji}$ when $i > j$. The symmetric matrix $\mathcal{N}$ represents the weighted adjacency matrix of the graph, whose elements are associated with the length n_{ij} of the edge.

The subsystem Σ_i of the system Σ we considered is captured by the following state-space equation:

$$\begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \\ z_i(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} A^i_{TT} & A^i_{Th} & A^i_{TS} & B^i_{Td} \\ A^i_{ST} & A^i_{Sh} & A^i_{SS} & B^i_{Sd} \\ C^i_{Tz} & C^i_{Th} & C^i_{Sz} & D^i_{zd} \\ C^i_{Ty} & C^i_{yh} & C^i_{Sy} & D^i_{yd} \end{bmatrix} \begin{bmatrix} x_i(t) \\ x_i(t-h_i) \\ v_i(t) \\ d_i(t) \end{bmatrix} \tag{1}$$

with the initial condition

$$x_i(t) = \phi_i(t), \quad t \in [-h_i, \ 0], \tag{2}$$

and the interconnection relationship

$$\begin{bmatrix} v_{ij}(t) \\ w_{ij}(t) \end{bmatrix} = \begin{bmatrix} w_{ji}(t) \\ v_{ji}(t) \end{bmatrix} \tag{3}$$

where $x_i(t) \in \mathbb{R}^{m_i}$ is the state vector of the i-th subsystem Σ_i, $\phi_i(t)$ is the initial condition with some given continuous function $\phi_i : [-h_i, 0] \to \mathbb{R}$, $d_i(t) \in \mathbb{R}^{d_i}$ is a disturbance acting on subsystem Σ_i, $z_i(t) \in \mathbb{R}^{z_i}$ is a performance output, $w_i(t), v_i(t) \in \mathbb{R}^{n_i}$ are the overall interconnection signals used by Σ_i, $w_i(t) = \mathrm{col}_{1\le j \le L} w_{ij}(t)$, $v_i(t) = \mathrm{col}_{1\le j \le L} v_{ij}(t)$, $u_i(t) \in \mathbb{R}^{p_i}$ is a control input, and $y_i(t) \in \mathbb{R}^{q_i}$ is a measured output. Fig.1 is an example of a system Σ with $L = 4$ subsystems $\Sigma_i, i = 1, 2, 3, 4$.

Based on the well-known results for the systems interconnected over an arbitrary graph without state time delay [7] and the stability definitions of delay systems [9], the following definitions can be given.

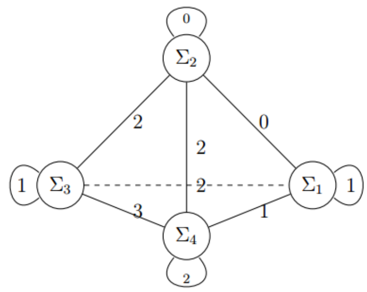

Figure 1. Example of the system Σ with $L = 4$ subsystems Σ_i, $i = 1, 2, 3, 4$.

Definition 1: The interconnected system Σ is well-posed if the two subspaces $\mathcal{I}$ and $\mathcal{B}$ satisfy

$$\mathcal{I} \cap \mathcal{B} = \{0\}, \tag{4}$$

where

$$\begin{aligned} \mathcal{I} &:= \{(w(t), v(t) \in \mathbb{R}^{\mathcal{N}} \times \mathbb{R}^{\mathcal{N}} : w_{ji}(t) = v_{ij}(t), \\ &\qquad \text{for all } \ i, j = 1, 2, \cdots, L\}, \\ \mathcal{B} &:= \{(w(t), v(t)) \in \mathbb{R}^{\mathcal{N}} \times \mathbb{R}^{\mathcal{N}} : \mathrm{col}(w_i(t), v_i(t)) \in \mathcal{B}^i, \\ &\qquad \text{for all } \ i, j = 1, 2, \cdots, L\}, \end{aligned} \tag{5}$$

with $\mathcal{B}_i := \mathrm{Im}\begin{bmatrix} A^i_{SS} \\ I \end{bmatrix}$.

Definition 2: The well-posed system Σ is stable with the disturbances $d_i(t) = 0, i = 1, 2, \ldots, L$, if, for any initial condition $\phi_i(t)$ defined on $[-h_i, 0]$, $i = 1, 2, \cdots, L$,

$$\lim_{t\to\infty} \sum_{i=1}^{L} x_i^*(t, \phi_i(t)) x_i(t, \phi_i(t)) = 0. \tag{6}$$

Definition 3: Assuming the system Σ is well-posed and stable, we say that it is contractive if, for all $d_i(t) \in \mathbb{R}^n, i = 1, 2, \cdots, L$ are non-identical zeros, there exist $\epsilon > 0$ such that

$$\sum_{i=1}^{L} \| z_i(t)\|_2^2 \le (1-\epsilon) \sum_{i=1}^{L} \| d_i(t)\|_2^2 \tag{7}$$

is satisfied under the zero initial condition.

III. Sufficient Conditions for Well-Posedness, Delay-Dependent Stability and Contractiveness

In this section, we shall present a sufficient condition to achieve well-posedness, delay-dependent stability and contractiveness with respected to the system Σ.

In this position we state our analysis conditions.

Theorem 1: The system Σ is well-posed, delay-dependent stable and contractive if there exist positive definite matrices $X_T^i \in \mathbb{R}_S^{m_i \times m_i}$, $R_i \in \mathbb{R}_S^{m_i \times m_i}$, $X_h^i \in \mathbb{R}_S^{m_i \times m_i}$ for $i = 1, \cdots, L$ and matrices $X_{ij}^{11} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ $X_{ij}^{12} \in \mathbb{R}^{n_{ij} \times n_{ij}}$ and $X_{ij}^{22} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ for all $i = 1, \cdots, L$ such that

$$X_{ij}^{11} = -X_{ji}^{22}, (X_{ij}^{12})^* = -X_{ji}^{12}, \tag{8}$$

and the following inequalities are satisfied for all $i = 1, 2, \cdots, L$

$$\Gamma_i^* \Psi_i \Gamma_i < 0, \tag{9}$$

where

$$\Psi_i := \begin{bmatrix} \Phi_i & X_T^i & \frac{1}{h_i} X_h^i & 0 & 0 & 0 & 0 \\ X_T^i & h_i X_h^i & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{h_i} X_h^i & 0 & M_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_i^{11} & X_i^{12} & 0 & 0 \\ 0 & 0 & 0 & (X_i^{12})^* & X_i^{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}, \tag{10}$$

$$\Gamma_i := \begin{bmatrix} I & 0 & 0 & 0 \\ A_{TT}^i & A_{Th}^i & A_{TS}^i & B_{Td}^i \\ 0 & I & 0 & 0 \\ A_{ST}^i & A_{Sh}^i & A_{SS}^i & B_{Sd}^i \\ 0 & 0 & I & 0 \\ C_T^i & C_{Th}^i & C_S^i & D^i \\ 0 & 0 & 0 & I \end{bmatrix}, \tag{11}$$

$$\Phi_i := R_i - \frac{1}{h_i} X_h^i, \;\; M_i := -R_i - \frac{1}{h_i} X_h^i \;, \tag{12}$$

$$X_i^{\bullet} := -\mathrm{diag}_{1 \le j \le L} X_{ij}^{\bullet}, \;\; \bullet \text{denote } 11,\ 12,\ 22 \;. \tag{13}$$

Proof: From the 3×3 block of matrix $\Gamma_i^* \Psi_i \Gamma_i$, we have

$$\begin{bmatrix} A_{SS}^i \\ I \end{bmatrix}^* \begin{bmatrix} X_i^{11} & X_i^{12} \\ (X_i^{12})^* & X_i^{22} \end{bmatrix} \begin{bmatrix} A_{SS}^i \\ I \end{bmatrix} < 0. \tag{14}$$

Furthermore, summing (14) over $i = 1, 2, \cdots, L$, we can get

$$\sum_{i=1}^{L} \begin{bmatrix} A_{SS}^i \\ I \end{bmatrix}^* \begin{bmatrix} X_i^{11} & X_i^{12} \\ (X_i^{12})^* & X_i^{22} \end{bmatrix} \begin{bmatrix} A_{SS}^i \\ I \end{bmatrix} < 0. \tag{15}$$

Again, since (8) is satisfied, we obtain

$$f(w(t), v(t)) := \sum_{i=1}^{L} \begin{bmatrix} w_i(t) \\ v_i(t) \end{bmatrix}^* \begin{bmatrix} X_i^{11} & X_i^{12} \\ (X_i^{12})^* & X_i^{22} \end{bmatrix} \begin{bmatrix} w_i(t) \\ v_i(t) \end{bmatrix}$$
$$= -\sum_{i=1}^{L} \sum_{j=1}^{L} \begin{bmatrix} w_{ij}(t) \\ v_{ij}(t) \end{bmatrix}^* \begin{bmatrix} X_{ij}^{11} & X_{ij}^{12} \\ (X_{ij}^{12})^* & X_{ij}^{22} \end{bmatrix} \begin{bmatrix} w_{ij}(t) \\ v_{ij}(t) \end{bmatrix} = 0. \tag{16}$$

Then, we have $\mathcal{I} \cap \mathcal{B} = \{0\}$, according to Definition 1, i.e., the interconnected system Σ is well-posed.

Since (9) is satisfied, pre- and post-multiplying (9) by the non-zero vector $g_i(t) := (x_i^*(t), v^*(t), d^*(t))$ and its transp-ose and summing over $i = 1, 2, \cdots, L$, we can obtain

$$\sum_{i=1}^{L} \langle 2x_i(t), X_T^i \dot{x}_i(t) \rangle + \sum_{i=1}^{L} \langle x_i(t), \Phi_i x_i(t) \rangle +$$
$$\sum_{i=1}^{L} \langle x_i(t - h_i), M_i x_i(t - h_i) \rangle + \sum_{i=1}^{L} \langle \dot{x}_i(t), h_i X_h^i \dot{x}(t) \rangle +$$
$$\sum_{i=1}^{L} \langle 2x_i(t), \frac{1}{h_i} X_h^i x_i(t - h_i) \rangle + f(w(t), v(t)) +$$
$$\sum_{i=1}^{L} z_i^*(t) z_i(t) - \sum_{i=1}^{L} d_i^*(t) d_i(t) < 0. \tag{17}$$

When $d_i(t) = 0$, and $f(w(t), v(t)) = 0$, (17) turns to

$$\sum_{i=1}^{L} \langle 2x_i(t), X_T^i \dot{x}_i(t) \rangle + \sum_{i=1}^{L} \langle x_i(t), \Phi_i x_i(t) \rangle$$
$$+ \sum_{i=1}^{L} \langle x_i(t - h_i), M_i x_i(t - h_i) \rangle + \sum_{i=1}^{L} \langle \dot{x}_i(t), h_i X_h^i \dot{x}(t) \rangle \tag{18}$$
$$+ \sum_{i=1}^{L} \langle 2x_i(t), \frac{1}{h_i} X_h^i x_i(t - h_i) \rangle < -\sum_{i=1}^{L} z_i^*(t) z_i(t) < 0.$$

We choose Lyapunov-Krasovskii function for the system

$$V_i(x_i(t), \phi_i(t)) = x_i^*(t) X_T^i x_i(t) + \int_{t-h_i}^{t} x_i^*(s) R_i x_i(s) ds$$
$$+ \int_{-h_i}^{0} \int_{t+\sigma}^{t} x_i^*(s) X_h^i x_i(s) ds d\sigma. \tag{19}$$

Since $X_T^i > 0$, $X_h^i > 0$, $R_i > 0$, $V_i(x, t) > 0$ is guaranteed. Besides, the derivatives of $\dot{V}_i(x, t)$, and summing over $i = 1, 2, \cdots, L$, are given by

$$\dot{V}(x, t) = \sum_{i=1}^{L} \dot{V}_i(x_i(t), \phi_i(t)) = 2\sum_{i=1}^{L} x_i^*(t) X_T^i \dot{x}_i(t)$$
$$+ \sum_{i=1}^{L} x_i^*(t) \Phi_i x_i(t) + \sum_{i=1}^{L} x_i^*(t - h_i) M_i x_i(t - h_i) \tag{20}$$
$$+ \sum_{i=1}^{L} \dot{x}_i^*(t) h_i X_h^i \dot{x}(t) \rangle + \sum_{i=1}^{L} x_i^*(t) \frac{1}{h_i} X_h^i x_i(t - h_i).$$

According the inequality (18), we have $\dot{V}(x, t) < 0$, which establishes the stability of the system.

When $d_i(t) \neq 0$, for all $i = 1, 2, \cdots, L$, along with the well-posedness and stability are proved, (17) yields

$$\dot{V}(x, t) + \sum_{i=1}^{L} z_i^*(t) z_i(t) + \sum_{i=1}^{L} d_i^*(t) d_i(t) < 0. \tag{21}$$

there must exist a positive scalar ϵ such that

$$\dot{V}(x, t) + \sum_{i=1}^{L} z_i^*(t) z_i(t) + \sum_{i=1}^{L} d_i^*(t) d_i(t) < -\epsilon \sum_{i=1}^{L} d_i^*(t) d_i(t). \tag{22}$$

Integrate (22) from 0 to ∞ under zero initial condition $\phi_i(t) = 0, t \in [-h_i, 0]$ for all $i = 1, 2, \cdots, L$, then we export that

$$\sum_{i=1}^{L} \int_0^{\infty} z_i^*(t) z_i(t) dt + (1 - \epsilon) \sum_{i=1}^{L} \int_0^{\infty} d_i^*(t) d_i(t) dt < -\int_0^{\infty} \dot{V}(x, t) dt,$$

that is to say

$$\| z(t) \|_2 - (1 - \epsilon) \| d(t) \|_2 < V(0) - V(\infty) = 0 \,, \tag{23}$$

which implies that the system Σ is contractive. This completes the proof.

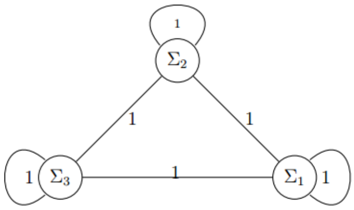

Figure 2. The system Σ with 3 subsystems $\Sigma_1, \Sigma_2, \Sigma_3$.

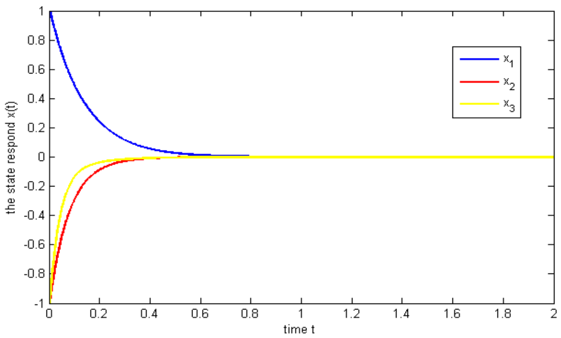

Figure 3. State responses of the system.

IV. Illustrative Example

In the following example, we consider the system Σ with three subsystems Σ_1, Σ_2, Σ_3 interconnected over an undirected graph as Fig. 2, and the state space matrices of each subsystem are given as follows:

$$\Sigma_1 = \begin{bmatrix} -7.267 & 0.574 & 9.528 & 4.077 & -1.448 & -0.712 \\ -0.829 & 0.0768 & -6.275 & 8.118 & -3.358 & -0.673 \\ 2.005 & 0.018 & 6.459 & 9.071 & -6.110 & 0.266 \\ 5.083 & 0.114 & -5.423 & 0.030 & -8.418 & 0.517 \\ -0.279 & -0.362 & -4.652 & 2.430 & 6.800 & -0.604 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} -5.476 & 0.285 & 8.089 & 7.327 & 8.451 & 0.127 \\ -6.274 & 0.036 & -6.396 & -4.974 & -1.358 & 0.439 \\ 3.532 & -0.737 & -1.731 & 3.300 & -8.084 & -0.233 \\ -2.854 & -0.141 & 6.019 & -6.461 & -3.981 & -0.123 \\ -9.818 & 0.987 & -6.549 & -8.336 & 3.103 & -0.130 \end{bmatrix}$$

$$\Sigma_3 = \begin{bmatrix} -5.813 & -0.170 & 6.779 & -9.026 & -6.404 & 0.406 \\ -4.043 & -0.172 & 5.433 & 4.360 & 7.718 & -0.674 \\ -1.535 & -0.254 & -5.240 & -5.623 & 9.991 & 0.254 \\ 0.675 & -0.515 & 2.512 & -0.214 & 5.731 & -0.347 \\ 1.991 & 0.015 & 4.972 & 7.251 & -8.769 & 0.215 \end{bmatrix}.$$

In the simulation, we assume that the state time delay of each subsystem is given by $h_1 = 1, h_2 = 2, h_3 = 1$, and the initial condition is given by $\phi_1(t) = 1, t \in [-1, 0]$, $\phi_2(t) = -1, t \in [-2, 0]$ and $\phi_3(t) = -1, t \in [-1, 0]$, respectively. Fig. 3 shows the state response of the system state $x(t)$, here $x(t) = \mathrm{col}_{1 \le i \le 3} x_i(t)$. Obviously the simulation result illustrates that the system Σ is stable. Fig. 4 shows that the scalar of the function:

$$f(t) = \sum_{i=1}^{L} \| z_i(t) \|_2^2 \Big/ \sum_{i=1}^{L} \| d_i(t) \|_2^2$$

is less than 1 over the time t under the disturbances $d_1(t) = 2e^{-2t}$, $d_2(t) = e^{-0.2t}$, $d_3(t) = e^{-0.04t}$, which exhibits that the system Σ is contractive.

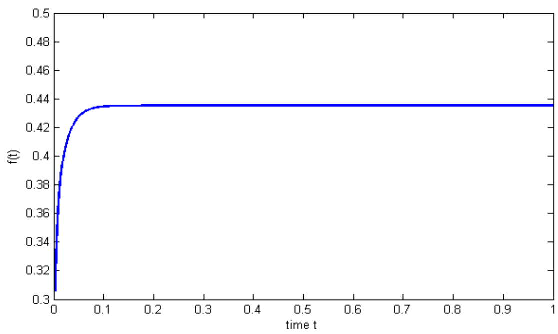

Figure 4. The performance of the system.

V. Conclusion

In this paper, we have developed a sufficient condition for the well-posedness, delay-dependent stability and contractiveness of the time delay system interconnected over an undirected graph in terms of linear matrix inequalities (LMIs). A numerical example has been given to demonstrate the validity of the obtained results. The next possible research topic is how to design distributed filters or controllers for such systems so that the resultant close-loop system or the filtering error system is well-posed, stable and contractive, when the measured output $y(t)$ inserts into another system (i.e. filters or controllers) as a control input or a disturbance.

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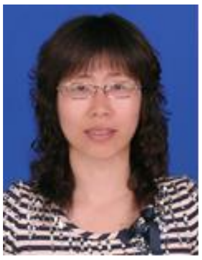

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