

Effective Relay Using Book of Beams in the NOMA Secondary Network

Hoai Trung Tran

University of Transport and Communications, Hanoi, Vietnam

Email: trungth@utc.edu.vn

Abstract—Non-Orthogonal Multiple Access (NOMA) technology is considered a strong candidate for 5G systems. The essence of this technique is to ensure that the near user can detect and subtract the far user's signal to get its signal for the detection. In addition, the system still provides enough signal strength for the far user. The article is interested in a 5G system that combines the primary and secondary networks, using an intermediate relay to support signals for the far secondary user. Several techniques involve inducing the transmit beamforming at the relay to produce the best signal at the far secondary user. Still, at the same time, it can generate the lowest interference to the primary; near secondary users and minimize its noise. These methods have some disadvantages, such as the complex optimal transmit beamforming algorithms or still not guaranteeing the smallest outage probability for the near and far users. The method proposed in this paper focuses on providing a book of beams combined with a hierarchical search algorithm to search for the productive beam faster, without complexity, and at the same time can minimize the outage probability thanks to the beamwidth that can be adjustable.

Index Terms—NOMA, beamforming, relay, multiple antenna, outage probability

I. INTRODUCTION

Non-Orthogonal Multiple Access (NOMA) is a strong candidate for a 5G cognitive system due to its ability to improve spectrum efficiency [1, 2]. Its working principle is dividing power according to the user's distance, where the far users are allocated higher power, decoded without Successive Interference Cancellation (SIC). In contrast, the users are closer to being assigned the lower power. However, it does use the SIC to subtract the far users' signals that have been previously decoded, resulting in significantly improved interference reduction [2–4]. It is worth noting that they all use the same total frequency band with sharing the variable transmit power of the secondary transmitter, which leads to improved performance compared to Orthogonal Multiple Access (OMA), which uses frequency separation for different users. In addition, it also provides high throughput, high power efficiency, and low latency [5, 6]. Moreover, the NOMA enhances spectrum utilization, spectral efficiency, and massive connectivity and has better fairness [7, 8]. It

also supports the increased number of Secondary Users (SUs) and their speeds [9].

In addition to the advantages of increased efficiency using spectrum for users, the aspects are of interest to NOMA to work efficiently. They consist of the power distribution at the Secondary Transmitter (ST) [10], maximization of the minimum lifetime of the SUs [11] or their minimum speeds thresholds [4], as well as the outage probability and the throughput for the SUs [12]. Maximizing the near user's throughput while the only threshold needed for the arrival signal strength at the far user is also considered [7]. The QoS parameters at the Primary User (PU) are the constraint addressed for the SU to operate in the underlay mode when sharing the same spectrum with the primary network [13].

Besides the above-considered aspects, some papers were also interested in a Relay (R). Firstly, they also clarify the role of the mobile as the R as it usually has a better channel that will use the SIC to decode the far mobile signals, then remove them from their received signal [5, 9, 14–16]. The far user signals are decoded at the relay mobile and forwarded to the corresponding mobiles, increasing the system's performance [9]. Similarly, the role of the relay mobile is also evident in [8], where it is described as an amplify-and-forward element. It enhances the signal transmitted from the Secondary Transmitter (ST) to the SUs. It increases the Signal to Interference and Noise Ratio (SINR) for the SUs, makes the SIC process more convenient at the R when using the NOMA technique, and reduces the interference from the ST and the R to the Primary Users (PUs) due to the low transmitting power to the SUs. In addition, the coverage of the ST through the R to the SUs is also broader. This technique is also called cooperation [17]. Secondly, they also consider using a dedicated relay, not mobile [6, 17–20] or one of a group of relays [3, 21, 22] for transmitting a signal to the mobiles. The signal can be from a Primary Transmitter (PT) such as [6, 19] or two primary and secondary transmitters such as [7, 21], and [23]. Therefore, we can see the importance of the R in the NOMA.

In the case of the dedicated relay, beamforming should be considered when it improves the network performance. It is interesting in the CVX shown in the paper [7, 12]. The CVX method is a MATLAB-based model system for convex optimization. The CVX is used to maximize the SU1's speed (objective) while making the SU2's speed exceed the threshold and interference of the ST and the

relay to the PU not to exceed the threshold (constraints). However, this complex algorithm requires extensive time to find the productive beam vector. Some articles are also interested in using interference suppression or the maximum gain beamforming techniques for the far users. Here, they consider using the Zero-Forcing Beam Forming (ZFBF). This method is a spatial signal processing method where a multi-antenna relay can give the null direction of the beam to other undesired users in the NOMA system, specifically introduced in articles [1], [24–26]. Articles [1] and [24] favor the use of the ZFBF at the R, and it also can artificially interfere with the eavesdroppers while enhancing the signal to the desired SU. Although [25] and [26] mention beamforming but do not state the R, they mainly focus on the transmitting beamforming of the Base Station (BS), the receiving beamforming of the PU (cell center), and the SU (cell edge). The primary purpose of these articles is to create the interference alignment matrix and find the received beamforming of the SUs against the intercell interference. The second step is to find the received beamforming of the PU so that they can cancel the interference of the BS station, which is transmitting to other PUs. Finally, the transmit beamforming at the BS to each user group is orthogonal to the channel matrices to the remaining groups and the interference alignment matrix caused by the intercell interference. This complicated job requires a lot of channel information between the BS, the PUs, and the SUs. In [27], the articles uses only one of multiple antennas for the BS, the far PU, and the near SU, not utilizing the beamforming and the R to achieve maximum speed at SU while ensuring a threshold for the received SINR at the PU together with the SINR at the SU to detect the PU signal. Using only one antenna wastes other antennas, and broadcasting the far PUs is challenging to extend without the R. Therefore, another solution is using the narrow beams, as shown in the proposed beams book. Finding the desired narrow beam can be found through the hierarchical search. This is the best search method that avoids affecting nearby users.

The problem is that the conventional methods for the relay usually choose the maximum beamforming TZF, avoid interference of other channels when broadcasting to distant users, or choose one of many beam candidates to make the largest signal-to-interference power ratio implemented. However, these methods all show that they still need to be interested in the beam width adjustment. This width change can make the relay reach distant secondary users while preventing interference with other users. The author has proposed a hierarchical beam method with layered beam width that can be changed according to the users' location.

II. SYSTEM MODEL

According to [7], we have a system model in which it is assumed that the relay has a transmit beamforming vector and a received beamforming vector, as illustrated in Fig. 1. The ST and the R cause interference to the PU using the shared bandwidth. In the case of an underlay, the ST and the R transmit signal on this band to the SU.

Thus they are also causing the interference above. To ensure good performance of the PU, the system designer usually sets a threshold level for the total noise from the ST and the R.

The received signal at the SU1 is [7]:

$$y_{SU1}[n] = h_{ST_SU1}s_{ST}[n] + \sqrt{P_R} \mathbf{h}_{R_SU1}^T \mathbf{w}_{T,R} x_{ST_SU2}[n - \tau] + n_1[n] \quad (1)$$

where h_{ST_SU1} is the fading from the ST to the SU1; $s_{ST}[n]$ is the signal from the ST; P_R is the power from the relay R. \mathbf{h}_{R_SU1} is the fading channel vector from the relay R to the SU1; $\mathbf{w}_{T,R}$ is the relay's transmit beamforming. $x_{ST_SU2}[n - \tau]$ is the information from the ST to the SU2, where τ is the propagation time; $n_1[n]$ is the Additive Gaussian White Noise (AGWN) signal at the SU1. $s_{ST}[n]$ is described as

$$s_{ST}[n] = \sqrt{P_S a_1} x_{ST_SU1}[n] + \sqrt{P_S a_2} x_{ST_SU2}[n] \quad (2)$$

where P_S is the transmit power from the ST; a_1 and a_2 are the transmit power allocation coefficients from the ST; $x_{ST_SU2}[n]$ is the information from the ST to the SU2.

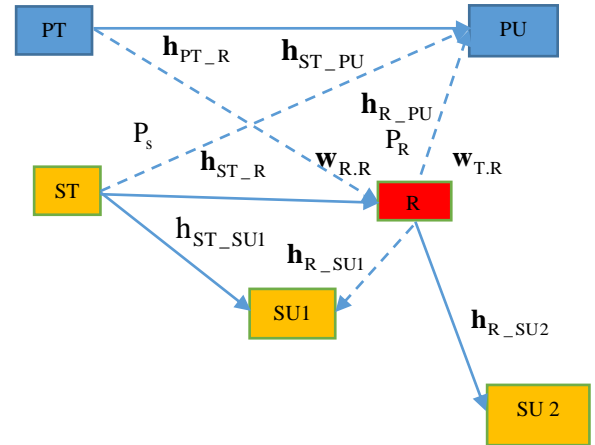


Fig. 1. Channel model.

The SINR of the SU2 at the SU1 is [7]:

$$\text{SNR}_{SU2 \text{ at } SU1} = \frac{P_S a_2 |h_{ST_SU1}|^2}{P_S a_1 |h_{ST_SU1}|^2 + P_R |\mathbf{h}_{R_SU1}^T \mathbf{w}_{T,R}|^2 + \sigma_n^2}. \quad (3)$$

And the SINR of the SU1 [2] is [7]:

$$\text{SNR}_{SU1} = \frac{P_S a_1 |h_{ST_SU1}|^2}{P_R |\mathbf{h}_{R_SU1}^T \mathbf{w}_{T,R}|^2 + \sigma_n^2} \quad (4)$$

where σ_n^2 is the AWGN noise power.

The received signal at the relay R is [7]:

$$y_R[n] = \mathbf{w}_{R,R}^H \mathbf{h}_{ST_R} s_{ST}[n] + \sqrt{P_R} \mathbf{w}_{R,R}^H \mathbf{h}_{R_R} \mathbf{w}_{T,R} x_{ST_SU2}[n - \tau] + \sqrt{P_{PT}} \mathbf{w}_{R,R}^H \mathbf{h}_{PT_R} x_{PT}[n] + \mathbf{w}_{R,R}^H n_R[n] \quad (5)$$

where $\mathbf{w}_{R,R}$ is the relay's receive beamforming; $\mathbf{h}_{ST,R}$ is the channel vector from the ST to the R. $\mathbf{H}_{R,R}$ is the interference matrix of the relay itself; $x_{ST,SU2}[n-\tau]$ is the signal from the ST to the SU2. P_{PT} is the transmit

power from the PT; $\mathbf{h}_{PT,R}$ is the channel vector from the PT to the relay; $x_{PT}[n]$ is the information from the PT; $n_r[n]$ is the AGWN signal at the relay R.

$$\text{SNR}_R = \frac{P_S a_2 |\mathbf{w}_{R,R}^H \mathbf{h}_{ST,R}|^2}{P_S a_1 |\mathbf{w}_{R,R}^H \mathbf{h}_{ST,R}|^2 + P_R |\mathbf{w}_{R,R}^H \mathbf{H}_{R,R} \mathbf{w}_{T,R}|^2 + P_U |\mathbf{w}_{R,R}^H \mathbf{h}_{PT,R}|^2 + \sigma_n^2 |\mathbf{w}_{R,R}^H|^2} \quad (6)$$

Therefore, the SINR at the relay is given in Eq. (6). The received signal at the SU2 is [7]:

$$y_{SU2} = \sqrt{P_R} \mathbf{h}_{R,SU2} \mathbf{w}_{T,R} x_{ST,SU2}[n-\tau] + n_{SU2}[n] \quad (7)$$

where $\mathbf{h}_{R,SU2}$ is the channel vector from the relay R to the SU2; $n_{SU2}[n]$ is the AGWN signal at the SU2.

The SINR at the SU2 is [7]:

$$\text{SNR}_{SU2} = \frac{P_R |\mathbf{h}_{R,SU2} \mathbf{w}_{T,R}|^2}{\sigma_n^2} \quad (8)$$

$$\max_{\mathbf{w}_{T,R}, \mathbf{w}_{R,R}, P_S, P_R} \log_2(1 + \text{SNR}_{SU2}) = \log_2 \left(1 + \frac{P_S a_1 |h_{ST,SU1}|^2}{P_R |\mathbf{h}_{R,SU1}^T \mathbf{w}_{T,R}|^2 + \sigma_n^2} \right) \quad (\text{max rate for the SU1}) \quad (9)$$

$$\frac{P_S a_2 |h_{ST,SU1}|^2}{P_S a_1 |h_{ST,SU1}|^2 + P_R |\mathbf{h}_{R,SU1}^T \mathbf{w}_{T,R}|^2 + \sigma_n^2} \geq \tilde{r} \quad (\text{rate of SU2 known by SU1}) \quad (10)$$

$$\frac{P_S a_2 |\mathbf{w}_{R,R}^H \mathbf{h}_{ST,R}|^2}{P_S a_1 |\mathbf{w}_{R,R}^H \mathbf{h}_{ST,R}|^2 + P_R |\mathbf{w}_{R,R}^H \mathbf{H}_{R,R} \mathbf{w}_{T,R}|^2 + P_P |\mathbf{w}_{R,R}^H \mathbf{h}_{PT,R}|^2 + |\mathbf{w}_{R,R}^H|^2 \sigma_n^2} \geq \tilde{r} \quad (\text{rate of SU2 at the relay}) \quad (11)$$

$$\frac{P_R |\mathbf{h}_{R,SU2} \mathbf{w}_{T,R}|^2}{\sigma_n^2} \geq \tilde{r} \quad (\text{rate at the SU2}) \quad (12)$$

Introducing the normalization:

$$|\mathbf{w}_{T,R}|^2 = |\mathbf{w}_{R,R}|^2 = 1 \quad (13)$$

The transmit powers of the ST and the relay R become

$$P_S = P_R > 0 \quad (14)$$

From Eq. (10), the relationship between relay's transmit beamforming $\mathbf{w}_{T,R}$ and the coefficient q_1 , that is concerned with the threshold \tilde{r} , is stated as [7]:

$$P_R |\mathbf{h}_{R,SU1}^T \mathbf{w}_{T,R}|^2 \leq \frac{P_S a_2 |h_{ST,SU1}|^2}{\tilde{r} - P_S a_1 |h_{ST,SU1}|^2 + \sigma_n^2} = q_1 \quad (15)$$

Similarly, (11) is equivalent to

$$\frac{P_S a_2 \mathbf{w}_{R,R}^H \mathbf{h}_{ST,R} \mathbf{h}_{ST,R}^H \mathbf{w}_{R,R}}{P_S a_2 \mathbf{w}_{R,R}^H \mathbf{A} \mathbf{w}_{R,R}} \geq \tilde{r} \quad (16)$$

Setting:

$$\begin{aligned} A &= P_S a_1 |\mathbf{h}_{ST,R}|^2 + P_R |\mathbf{H}_{R,R} \mathbf{w}_{T,R}|^2 + P_{PT} |\mathbf{h}_{PT,R}|^2 + \sigma_n^2 \\ &= P_R |\mathbf{H}_{R,R} \mathbf{w}_{T,R}|^2 + B \end{aligned}$$

Next, we maximize the rate for the SU1. Meanwhile, the data rate of the SU2 observed by the SU1, the relay must be above the required threshold. The rate of the SU2 for itself must also be above this threshold. In addition, it is necessary to specify that the normalization function of the transmit and receive beamforming vectors is 1. The powers from the ST and the R have positive values.

The exercise is given in Eq. (9). With constraints on SINR threshold of the SU2, for \tilde{r} we have the relationships of Eq. (10) to Eq. (12).

where $B = P_S a_1 |\mathbf{h}_{ST,R}|^2 + P_{PT} |\mathbf{h}_{PT,R}|^2 + \sigma_n^2$, in which P_{PT} is the transmit power from the PT.

Eq. (16) becomes

$$\begin{aligned} \mathbf{h}_{ST,R} \left(P_R |\mathbf{H}_{R,R} \mathbf{w}_{T,R}|^2 + B \right)^{-1} \mathbf{h}_{ST,R}^H &= \mathbf{h}_{ST,R} B^{-1} \mathbf{h}_{ST,R}^H - \\ &\frac{(\mathbf{h}_{ST,R}^H B^{-1} \mathbf{H}_{R,R} \mathbf{w}_{T,R} \mathbf{w}_{T,R}^H \mathbf{H}_{R,R}^H B^{-1} \mathbf{h}_{ST,R}) P_R}{1 + P_R \mathbf{w}_{T,R}^H \mathbf{H}_{R,R}^H B^{-1} \mathbf{H}_{R,R} \mathbf{w}_{T,R}} \geq \frac{\tilde{r}}{P_S a_2} \end{aligned} \quad (17)$$

with the optimum beamforming vector:

$$\mathbf{w}_{R,R} = \frac{\mathbf{A}^{-1} \mathbf{h}_{ST,R}}{\|\mathbf{A}^{-1} \mathbf{h}_{ST,R}\|} \quad (18)$$

Therefore,

$$\begin{aligned} \frac{(\mathbf{h}_{ST,R}^H B^{-1} \mathbf{H}_{R,R} \mathbf{w}_{T,R} \mathbf{w}_{T,R}^H \mathbf{H}_{R,R}^H B^{-1} \mathbf{h}_{ST,R}) P_R}{1 + P_R \mathbf{w}_{T,R}^H \mathbf{H}_{R,R}^H B^{-1} \mathbf{H}_{R,R} \mathbf{w}_{T,R}} &\leq \\ \mathbf{h}_{ST,R} B^{-1} \mathbf{h}_{ST,R}^H - \frac{\tilde{r}}{P_S a_2} &= q_2 \end{aligned} \quad (19)$$

Based on Eq. (12), we have

$$P_R \left| \mathbf{h}_{R_SU2} \mathbf{w}_{T,R} \right|^2 \geq \tilde{r} \sigma_n^2 = q_3 \quad (20)$$

Consider the interference from the ST and the SU to the PU (the total interference threshold I_{th}) [7]:

$$P_S \left| \mathbf{h}_{ST_PU} \right|^2 + P_R \left| \mathbf{h}_{R_PU}^T \mathbf{w}_{T,R} \right|^2 \leq I_{th} \quad (21)$$

$$P_R \left| \mathbf{h}_{R_PU}^T \mathbf{w}_{T,R} \right|^2 \leq I_{th} - P_S \left| \mathbf{h}_{ST_PU} \right|^2 = q_4 \quad (22)$$

Supposing P_S is a constant, what we need to do is choose $\mathbf{w}_{T,R}^H$ for Eq. (9) to maximize the max rate for the SU1. This means we need to make the minimum denominator $P_R \left| \mathbf{h}_{R_SU1}^T \mathbf{w}_{T,R} \right|^2 + \sigma_n^2$ in Eq. (9). Typically, σ_n^2 is assumed to be predetermined, so the remaining part of the denominator can be described as

$$\min_{\mathbf{w}_{T,R}} \bar{\mathbf{w}}_{T,R}^H \mathbf{h}_{R_SU1}^* \mathbf{h}_{R_SU1}^T \bar{\mathbf{w}}_{T,R} \quad (23)$$

where $\bar{\mathbf{w}}_{T,R} = \sqrt{P_R} \mathbf{w}_{T,R}$.

Similarly, from Eq. (15) we have [7]:

$$\bar{\mathbf{w}}_{T,R}^H \mathbf{h}_{R_SU1}^* \mathbf{h}_{R_SU1}^T \bar{\mathbf{w}}_{T,R} \leq q_1 \quad (24)$$

Eq. (19) is equivalent to

$$\begin{aligned} & \mathbf{h}_{ST_R}^H B^{-1} \mathbf{H}_{R_R} \bar{\mathbf{w}}_{T,R} \bar{\mathbf{w}}_{T,R}^H \mathbf{H}_{R_R}^H B^{-1} \mathbf{h}_{ST_R} \\ & \leq q_2 \left(1 + \bar{\mathbf{w}}_{T,R}^H \mathbf{H}_{R_R}^H B^{-1} \mathbf{H}_{R_R} \bar{\mathbf{w}}_{T,R} \right) \end{aligned} \quad (25)$$

And Eq. (20) can be turned as [7]:

$$\bar{\mathbf{w}}_{T,R}^H \mathbf{h}_{R_SU2}^* \mathbf{h}_{R_SU2}^T \bar{\mathbf{w}}_{T,R} \geq q_3 \quad (26)$$

Eq. (22) can be analyzed further [7]:

$$\bar{\mathbf{w}}_{T,R}^H \mathbf{h}_{R_PU}^* \mathbf{h}_{R_PU}^T \bar{\mathbf{w}}_{T,R} \leq q_4 \quad (27)$$

Let us move on to the problem using the trace operator [7]:

$$\min_{\bar{\mathbf{w}}_{T,R}} \text{tr} \bar{\mathbf{w}}_{T,R} \mathbf{h}_{R_SU1}^* \mathbf{h}_{R_SU1}^T \quad (28)$$

where $\bar{\mathbf{W}}_{T,R} = \mathbf{w}_{T,R} \mathbf{w}_{T,R}^H$.

Using the trace operator to other Eqs. (24)-(27), Eqs. (29)-(32) become correspondingly:

$$\text{tr} \left(\bar{\mathbf{W}}_{T,R} \mathbf{h}_{R_SU1}^* \mathbf{h}_{R_SU1}^T \right) \leq q_1 \quad (29)$$

$$\begin{aligned} & \text{tr} \left(\bar{\mathbf{W}}_{T,R} \mathbf{H}_{R_R}^H B^{-1} \mathbf{h}_{ST_R} \mathbf{h}_{ST_R}^H B^{-1} \mathbf{H}_{R_R} \right) \\ & \leq q_2 \left(1 + \text{tr} \left(\bar{\mathbf{W}}_{T,R} \mathbf{H}_{R_R}^H B^{-1} \mathbf{H}_{R_R} \right) \right) \end{aligned} \quad (30)$$

$$\text{tr} \left(\bar{\mathbf{W}}_{T,R} \mathbf{h}_{R_SU2}^* \mathbf{h}_{R_SU2}^T \right) \geq q_3 \quad (31)$$

$$\text{tr} \left(\bar{\mathbf{W}}_{T,R} \mathbf{h}_{R_PU}^* \mathbf{h}_{R_PU}^T \right) \leq q_4 \quad (32)$$

III. OUTAGE PROBABILITY

User SU1: Regarding the problem related to the outage probability, we are interested in the SU1 when there are two cases [7].

Case 1 - \mathbf{O}_{SU1}^1 when the SU1 cannot detect the signal of the user SU2, it cannot perform the next step of decoding.

Case 2 - \mathbf{O}_{SU1}^2 then the SU1 has detected x_2 as the user SU2's signal but failed to decode its signal x_1 .

Applying [7], they use two quantities, θ_1 and θ_2 , the signal-to-noise ratio of the SU1 and the SU2, respectively. If the speeds of the SU1 and the SU2 are R_1 and R_2 , then we have:

$$\theta_1 = 2^{R_1} - 1 \text{ and } \theta_2 = 2^{R_2} - 1 \quad (33)$$

Assuming the ST's power allocation for the SU1, the SU2 with the ratio b_0 and b_1 , we give two parameters:

$$Y_1 = \frac{\left| h_{ST_SU1} \right|^2}{\left| h_{ST_PU} \right|^2} \text{ and } Y_2 = \frac{\left| \mathbf{h}_{R_SU1} \mathbf{w}_{T,R} \right|^2}{\left| \mathbf{h}_{R_PU} \mathbf{w}_{T,R} \right|^2} \quad (34)$$

\mathbf{O}_{SU1}^1 : the SINR of the SU2 detected by the SU1 is below [7]:

$$\frac{b_0 Y_1}{b_1 Y_1 + b_2 Y_2 + b_3} \geq \theta_2 \quad (35)$$

\mathbf{O}_{SU1}^2 : the SINR of the SU1 after omitting the SU2 signal is [7]:

$$\frac{b_1 Y_1}{b_2 Y_2 + b_3} \geq \theta_1 \quad (36)$$

The outage probability at the SU1 is

$$P_{out,SU1} = P_r \left(Y_1 \leq \zeta (b_2 Y_2 + b_3) \right) \quad (37)$$

where $\zeta = \theta_1 / b_1$ or $\theta_2 / (b_0 - \theta_2 b_1)$.

Choosing $\zeta = \max(\theta_1 / b_1, \theta_2 / (b_0 - \theta_2 b_1))$. Here $\theta_1 = \text{SNR}_{SU1}$ from Eq. (4); $\theta_2 = \text{SNR}_{SU2 \text{ at } SU1}$ from Eq. (3). There are two cases: $\theta_2 > b_0 / b_1$ when $P_{out,SU1} = 1$;

$0 < \theta_2 < b_0 / b_1$ when $P_{out,SU1} = \int_0^\infty F_{Y_1}(\zeta(b_2 y + b_3)) f_{Y_2}(y) dy$.

We compute the components of the integral of the above expression. The first component (the cumulative distribution function of $Y_1 - F_{Y_1}(\cdot)$) based on [7] is

$$\begin{aligned} F_{Y_1}(\zeta(b_2 y + b_3)) &= \left(\frac{1}{\zeta(b_2 y + b_3) + 1} \right)^{-1} \\ &= \left(\frac{1 + \zeta(b_2 y + b_3)}{\zeta(b_2 y + b_3)} \right)^{-1} = \frac{\zeta(b_2 y + b_3)}{1 + \zeta(b_2 y + b_3)} \end{aligned} \quad (38)$$

The remaining component (the probability density function of the cumulative distribution function of $Y_2 - f_{Y_2}(y)$) is the gradient of $F_{Y_2}(\cdot)$ [7]:

$$F_{Y_2}(y) = \left(\frac{1}{y} + 1 \right)^{-1} = \left(\frac{1+y}{y} \right)^{-1} = \frac{y}{1+y} \quad (39)$$

We have the outage probability at the SU1 after replacing the components in the expression $P_{out,SU1}$ with Eq. (38) and Eq. (39):

$$P_{\text{out,SU1}} = 1 - \int_0^\infty \frac{1}{\varsigma(b_2 y + b_3) + 1} \left(\frac{1}{1+y} \right)^2 dy \quad (40)$$

$$= 1 - \int_0^\infty \frac{1}{\varsigma b_2 (y + (b_3 \varsigma + 1)/b_2 \varsigma)} \left(\frac{1}{1+y} \right)^2 dy$$

According to [29]:

$$\int_0^\infty x^{v-1} (\beta + x)^{-\mu} (x + \gamma)^{-\vartheta} dx \quad (41)$$

$$= \beta^{-\mu} \gamma^{v-\vartheta} B(v, \mu - v + \vartheta) {}_2F_1(\mu, v; \mu + \vartheta; 1 - \gamma/\beta)$$

where $v=1$, $\beta = \frac{b_3 \rho_{\text{th}} \varsigma + \rho_{\text{th}}}{b_2 \rho_{\text{th}} \varsigma} = \frac{2\varsigma + \rho_{\text{th}}}{\varsigma b_2 \rho_{\text{th}}}$, $\mu=1$, $\vartheta=2$,

with $\rho_{\text{th}} = I_{\text{th}}/\sigma_n^2$. Eq. (40) can be rewritten as

$$P_{\text{out,SU1}} = 1 - \frac{1}{2} \frac{\rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} {}_2F_1\left(1, 1; 3; 1 - \frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}}\right) \quad (42)$$

where ${}_2F_1$ is the hypergeometric function and has the value based on [30]:

$${}_2F_1(1, 1; m; z) = \frac{(m-1)z}{(z-1)^2} \left(\sum_{k=2}^{m-1} \frac{1}{m-k} \left(\frac{z-1}{z} \right)^k - \left(\frac{z-1}{z} \right)^m \log(1-z) \right) \quad (43)$$

Component 1 and Component 2 in the right of Eq. (43) are calculated by Eq. (44) and Eq. (45). Combining Eq. (43) – Eq. (45), Eq. (42) becomes Eq. (46).

$$\frac{(m-1)z}{(z-1)^2} \sum_{k=2}^{m-1} \frac{1}{m-k} \left(\frac{z-1}{z} \right)^k = \frac{(3-1) \left(1 - \frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)}{\left(-\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)^2} \frac{1}{3-2} \frac{\left(-\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)^2}{\left(1 - \frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)^2} = 2 \frac{2\varsigma + \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}} - \varsigma b_2 \rho_{\text{th}}} \quad (44)$$

$$\frac{(m-1)z}{(z-1)^2} \left[-\left(\frac{z-1}{z} \right)^m \log(1-z) \right] = \frac{(3-1) \left(1 - \frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)}{\left(-\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)^2} \left[-\left(\frac{\left(-\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)^3}{\left(1 - \frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)^3} \right) \log\left(\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right) \right] \quad (45)$$

$$= 2 \frac{\left(\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)}{\left(1 - \frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)^2} \log\left(\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right) = 2 \frac{\varsigma b_2 \rho_{\text{th}} (2\varsigma + \rho_{\text{th}})}{(2\varsigma + \rho_{\text{th}} - \varsigma b_2 \rho_{\text{th}})^2} \log\left(\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)$$

$$P_{\text{out,SU1}} = 1 - \frac{1}{2} \frac{\rho_{\text{th}}}{(2\varsigma + \rho_{\text{th}})^2} \left[2 \frac{2\varsigma + \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}} - \varsigma b_2 \rho_{\text{th}}} - 2 \frac{\varsigma b_2 \rho_{\text{th}} (2\varsigma + \rho_{\text{th}})}{(2\varsigma + \rho_{\text{th}} - \varsigma b_2 \rho_{\text{th}})^2} \log\left(\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right) \right] \quad (46)$$

$$= 1 - \frac{\rho_{\text{th}}}{2\varsigma + \rho_{\text{th}} - \varsigma b_2 \rho_{\text{th}}} + \frac{\varsigma b_2 \rho_{\text{th}}^2}{(2\varsigma + \rho_{\text{th}} - \varsigma b_2 \rho_{\text{th}})^2} \log\left(\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right)$$

$$= 1 - \frac{\rho_{\text{th}}}{2\varsigma + \rho_{\text{th}} - \varsigma b_2 \rho_{\text{th}}} \left(1 + \frac{\varsigma b_2 \rho_{\text{th}}}{(2\varsigma + \rho_{\text{th}} - \varsigma b_2 \rho_{\text{th}})} \log\left(\frac{\varsigma b_2 \rho_{\text{th}}}{2\varsigma + \rho_{\text{th}}} \right) \right)$$

$$= 1 - \frac{1}{2\varsigma/\rho_{\text{th}} + (1 - \varsigma b_2)} \left(1 + \frac{\varsigma b_2}{2\varsigma/\rho_{\text{th}} + (1 - \varsigma b_2)} \log\left(\frac{\varsigma b_2}{2\varsigma/\rho_{\text{th}} + 1} \right) \right)$$

The user SU2: for the outage probability for the SU2 is [7]:

$$P_{\text{out,SU2}} = P(\mathbf{O}_{\text{SU2}} \cup \mathbf{O}_{\text{SU2}}^2) \quad (47)$$

In this case, \mathbf{O}_{SU2} is the event where the R can not decode the signal x_2 ; $\mathbf{O}_{\text{SU2}}^2$ is the event where the SU2 can not decode x_2 .

Specifically, the outage probability for SU2 can be described:

$$P_{\text{out,SU2}} = F_{\gamma_R^{\text{TP}}} (z) = P_r \left(\frac{c_0 X_1}{c_1 X_1 + c_2 X_2 + 2/\rho_{\text{th}}} < z \right) \quad (48)$$

where $c_0 = a_2 (\lambda_{\text{ST-R}}/\lambda_{\text{PT-R}})$, $c_1 = a_1 (\lambda_{\text{ST-R}}/\lambda_{\text{PT-R}})$, $c_2 = 2\lambda_{\text{PT-R}} (\rho_{\text{PT}}/\rho_{\text{th}})$; $\rho_{\text{PT}} = P_{\text{PT}}/\rho_{\text{th}}$, in which $\lambda_{\text{PT-R}}$, $\lambda_{\text{ST-R}}$ are the channel gains from the PT and the ST to the R, respectively; and

$$X_1 = \frac{\|\mathbf{h}_{\text{ST-R}}\|^2}{\|\mathbf{h}_{\text{ST-PU}}\|^2}, \quad X_2 = \frac{\|\mathbf{h}_{\text{ST-R}}^H \mathbf{h}_{\text{PT-R}}\|^2}{\|\mathbf{h}_{\text{ST-R}}\|^2}.$$

The Eq. (48) is equivalent to:

$$F_{\gamma_R}^{\text{TZF}}(z) = P_r \left(X_1(c_0 - zc_1) < z \left(c_2 X_2 + \frac{2}{\rho_{\text{th}}} \right) \right)$$

$$= \begin{cases} \int_0^\infty F_{X_1} \left(\frac{z(\rho_{\text{th}} c_2 x + 2)}{\rho_{\text{th}}(c_0 - zc_1)} \right) f_{X_2}(x) dx & 0 < z < \frac{c_0}{c_1} \\ 1 - \int_0^\infty F_{X_1} \left(\frac{z(\rho_{\text{th}} c_2 x + 2)}{\rho_{\text{th}}(c_0 - zc_1)} \right) f_{X_2}(x) dx & z > \frac{c_0}{c_1} \end{cases} \quad (49)$$

where the cumulative distribution function of X_1 - $F_{X_1}(\cdot)$ [7] is

$$F_{X_1} \left(\frac{z(\rho_{\text{th}} c_2 x + 2)}{\rho_{\text{th}}(c_0 - zc_1)} \right)$$

$$= \begin{cases} \int_0^\infty \left(\frac{\rho_{\text{th}}(c_0 - zc_1)}{z(\rho_{\text{th}} c_2 x + 2)} + 1 \right)^{-N_R} e^{-x} dx & 0 < z < c_0/c_1 \\ 1 & z > c_0/c_1 \end{cases} \quad (50)$$

The remaining component (the probability density function of $X_2 - f_{X_2}(y)$) is the gradient of the cumulative distribution function $F_{X_2}(\cdot)$ [7]:

$$f_{X_2}(x) = \left(\frac{c_3}{z} + 1 \right)^{1-N_T} \text{ with } c_3 = \frac{\rho_{\text{th}}}{2} \quad (51)$$

N_R and N_T are the number of the relay R's receive and transmit antennas. After replacing Eq. (50) and Eq. (51) with Eq. (49), we have the outage probability of SU2 is

$$P_{\text{out}, \text{SU2}} = \begin{cases} \left(\left(\frac{c_3}{\theta_2} + 1 \right)^{1-N_T} + \left(1 - \left(\frac{c_3}{\theta_2} + 1 \right)^{1-N_T} \right) \right) \times \int_0^\infty \left(\frac{\rho_{\text{th}}(c_0 - zc_1)}{z(\rho_{\text{th}} c_2 x + 2)} + 1 \right)^{-N_R} e^{-x} dx, & \theta_2 < \frac{a_2}{a_1} \\ 1, & \theta_2 > \frac{a_2}{a_1} \end{cases} \quad (52)$$

IV. PROPOSED METHOD

We solve problem Eq. (28) with constraints from Eq. (29) to Eq. (32) using the CVX method. Secondly, another technique is involved the orthogonalization of the transmitted beam vector of the relay to devices such as the PU or the SU1. It should take into account the interference of the PT and the self-interference matrix at the relay while enhancing the rate to SU2 based on idea from [28]:

$$\max_{\mathbf{w}_{\text{T,R}}} \mathbf{w}_{\text{T,R}}^H \mathbf{h}_{\text{R,SU2}} \quad (53)$$

The constraints with the transmit relay beamforming is

$$\begin{bmatrix} \mathbf{h}_{\text{R,PU}}^H & \mathbf{h}_{\text{R,SU1}}^H \end{bmatrix} \mathbf{w}_{\text{T,R}} = 0 \quad (54)$$

Moreover

$$\max_{\mathbf{w}_{\text{T,R}}} \mathbf{w}_{\text{T,R}}^H \mathbf{h}_{\text{ST,R}} \quad (55)$$

The constraint with the receive relay beamforming is

$$\begin{bmatrix} \mathbf{h}_{\text{PT,R}}^H & \mathbf{H}_{\text{R,R}} \mathbf{w}_{\text{T,R}} \end{bmatrix} \mathbf{w}_{\text{R,R}} = 0 \quad (56)$$

Thirdly, a method, called the Transmit Zero Forcing (TZF) [7], is used to maximize the below formula:

$$\max_{\|\mathbf{w}_{\text{T,R}}\|=1} |\mathbf{h}_{\text{R,SU2}}^T \mathbf{w}_{\text{T,R}}|^2 \quad (57)$$

With condition:

$$\mathbf{h}_{\text{ST,R}}^H \mathbf{H}_{\text{R,R}} \mathbf{w}_{\text{T,R}} = 0 \quad (58)$$

From (57) and (58), we obtain

$$\mathbf{w}_{\text{T,R}} = \frac{\mathbf{B} \mathbf{h}_{\text{R,SU2}}^*}{\|\mathbf{B} \mathbf{h}_{\text{R,SU2}}^*\|} \quad (59)$$

where

$$\mathbf{B} = \mathbf{I} - \frac{\mathbf{H}_{\text{SI}}^H \mathbf{h}_{\text{ST,R}} \mathbf{h}_{\text{ST,R}}^H \mathbf{H}_{\text{SI}}}{\|\mathbf{h}_{\text{ST,R}}^H \mathbf{H}_{\text{SI}}\|^2}$$

The proposed method we use is the beams book, where the number of beam sets equals the number of antennas. This faster method to find the optimal transmit beam at the R is choosing one more productive beam of the two beams if the two transmit antennas are utilized [29]. If the number of antennas is larger, for example, 4, we can use a hierarchical search to find from the two beams of the first layer a better beam to increase the maximum speed from the relay to the SU2, for example, beam 1. We will split beam one into two sub-beams, one of which makes the SU2's speed higher to be selected as the final optimal transmit beam [31]. The larger the number of antennas, the larger the number of beams in the beams book. The beam width is narrower, making the relay more likely to direct the beam to the SU2, and it will be more difficult to interfere with the PU and the SU1. This system uses a set of beams with different beam widths for each layer to make beam generation more detailed. In addition, between the relay and the far, near secondary and primary users, it is necessary to have a feedback channel to ensure that the relay can know the location of these users. The relay can select beams and their layer index in the hierarchical book of beams.

V. SIMULATION

We can use the diagram in Fig. 1 to perform simulations in the four cases mentioned above. Initial assumptions: $\lambda = 0.01$ m; the PT, the ST, the SU1, and the SU2 are all equipped with one antenna; the R is equipped with two transmit and receive antennas, in which the distance between the two antennas: $s_T = s_R = 0.5$ m.

$$\begin{aligned}
 \angle_{ST_SU1} &= 286.5^\circ, |h_{ST_SU1}| = 1 \\
 \angle_{PT_R} &= -45^\circ, |h_{PT_R}| = 0.25 \\
 \angle_{ST_R} &= 0^\circ, |h_{ST_R}| = 0.5 \\
 \angle_{R_PU} &= 45^\circ, |h_{R_PU}| = 0.25 \\
 \angle_{R_SU2} &= -45^\circ, |h_{R_SU2}| = 0.5 \\
 \angle_{R_SU1} &= 206.5^\circ, |h_{R_SU1}| = 0.5 \\
 \angle_{ST_PU} &= 26.5^\circ, |h_{ST_PU}| = 0.25
 \end{aligned} \tag{60}$$

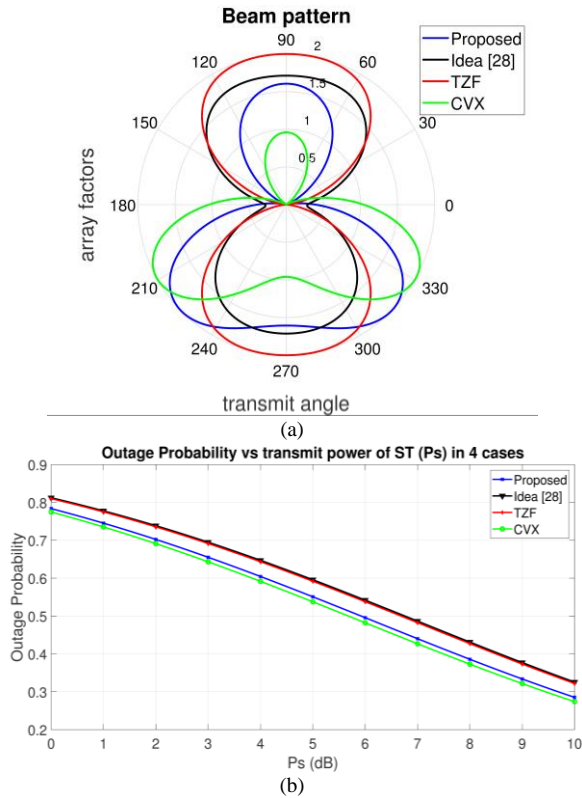


Fig. 2. The relay's performance: (a) Transmit beamforming at the SU1 and (b) outage probability at the SU1.

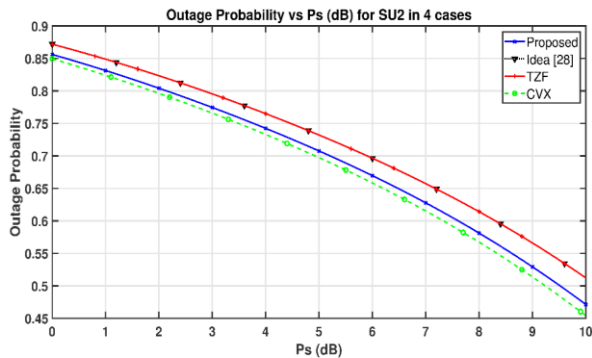


Fig. 3. The outage probability at the SU2.

We simulate the outage probability for the near user SU1 Eq. (40), using beamforming vectors in 4 cases: the CVX, the idea from [28], the TZF beamforming, and finally, the proposed hierarchical beam search (Fig. 2 (a) for beams and Fig. 2 (b) for the outage probabilities).

In the case of calculating the outage probability at the SU2, Fig. 3 describes the relationship between the outage

probability and the transmit power of the ST in the above cases. The equation for the outage probability is from Eq. (48).

If we look at the two figures above, we can see that the conventional method of the article [28] or the TZF method has an outage probability higher than the proposed method, not only at the SU1 but also at the SU2. The proposed method has a slightly higher outage probability than the CVX method. This is because the number of beams created from the number of antenna elements is not high (the number of transmitting antennas of the relay is 2). Hence, its beamwidth is not narrow enough to guarantee a lower error probability. However, the beam search in the proposed method will be much faster than the CVX method.

The new method uses a finite number of beams, whereas CVX selects beams among many beams (that makes the CVX more complex than the proposed method), resulting in longer beam search times.

On the other hand, CVX is an ideal case, so the new method can still achieve a low outage probability like CVX if the beams in the new method are chosen to coincide with the beams of CVX. If we make the beam width smaller in the new method, we can get the OP at SU1 equivalent to CVX. The new beam is described in Fig. 5.

When we have the beam on, we see the outage probability is equivalent to the case CVX. It is illustrated in Fig. 6.

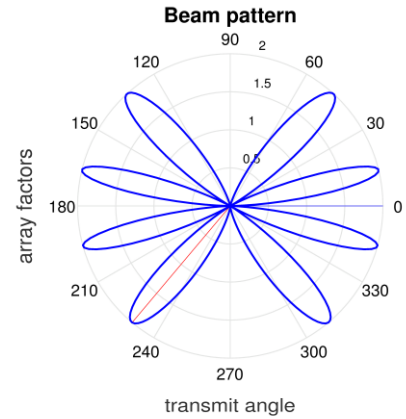


Fig. 5. Change the beamwidth in the proposed method

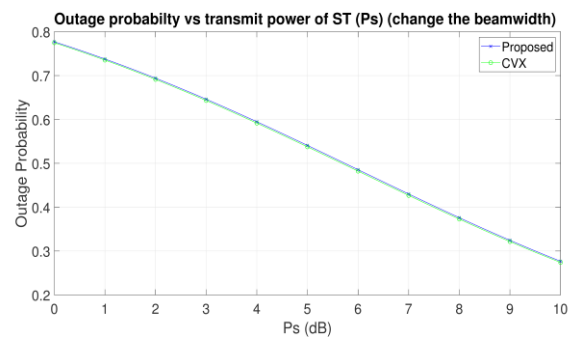


Fig. 6. The equivalence between the proposed method and the CVX when changing the beam width.

VI. CONCLUSION

The article explores the application of the book of beams and the hierarchical search algorithm in the 5G

system consisting of the combined primary and secondary network. Using the proposed beams book method reduces the productive beam finding time and, simultaneously, can create a low outage probability compared with other methods. This helps to optimize the beams book generation at the relay, which provides quality assurance for the far users while not affecting the performance of the primary or the near secondary users. Deploying the relay combined with the beams book using the multiple antennas is seen as a goal in the future when high-speed multi-user services are provided.

CONFLICT OF INTEREST

The author declares no conflict of interest.

ACKNOWLEDGMENT

The author wishes to thank the University of Transport and Communications, Hanoi, Vietnam, for supporting this paper.

REFERENCES

- [1] S. Majhi and P. Mitra, "A review of cognitive radio inspired NOMA techniques," *A EasyChair Preprint*, no. 2711, Feb. 2020.
- [2] H. Liao, "Capacity analysis of secondary user system in cognitive MIMO networks based on NOMA," *Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering book series*, vol. 237, pp. 426–439, 2018.
- [3] H. Liao, "Error performance of NOMA-based cognitive radio networks with partial relay selection and interference Polr," *IEEE Trans. on Communications*, vol. 68, no. 2, pp. 765–777, Feb. 2020.
- [4] L. Xu, H. Xing, Y. Deng, A. Nallanathan, and C. Zhuansun, "Fairness-Aware throughput maximization for underlaying cognitive NOMA networks," *IEEE Systems Journal*, vol. 15, no. 2, pp. 1881–1892, June 2021.
- [5] N. S. Kim, "Cooperative overlay cognitive radio NOMA network with channel errors and imperfect SIC," *A Int. Journal of Intelligent Engineering and Systems.*, vol. 12, no. 5, pp. 224–231, 2019.
- [6] D. T. Do, C. B. Le, and A. T. Le, "Cooperative underlay cognitive radio assisted NOMA: Secondary network improvement and outage performance," *Telkomnika*, vol. 17, no. 5, pp. 2147–2154, 2019.
- [7] M. Mohammadi, B. K. Chalise, A. Hakimi, *et al.*, "Beamforming design and power allocation for full-duplex non-orthogonal multiple access cognitive relaying," *IEEE Trans. on Communications*, vol. 66, no. 12, pp. 5912–5965, Dec. 2018.
- [8] L. Lv, J. Chen, Q. Ni, *et al.*, "Cognitive non-orthogonal multiple access with cooperative relaying: A new wireless frontier for 5G spectrum sharing," *IEEE Communications Magazine*, vol. 56, no. 4, pp. 188–195, April, 2018.
- [9] Z. Ali, W. U. Khan, G. A. S. Sidhu, *et al.*, "Fair power allocation in cooperative cognitive systems under NOMA transmission for future IoT networks," *Alexandria Engineering Journal*, vol. 61, pp. 575–583, 2022.
- [10] S. S. Abidrabu and H. Arslan, "Efficient pair allocation for cognitive radio NOMA using game-theoretic based pricing strategy," in *Proc. IEEE 93rd Vehicular Technology Conference (VTC2021-Spring)*, 2021.
- [11] N. Tang, H. Tang, T. L. Ngoc, B. Li, and X. Yuan, "Lifetime maximization for UAV-Enabled Cognitive-NOMA IoT networks: Joint location, pair, and decoding order optimization," *arXiv:2110.01133v2*, Oct. 2021.
- [12] T. N. Tran, D. T. Do, and M. Voznak, "Full-duplex cognitive radio NOMA networks: Outage and throughput performance analysis," *Intl Journal Of Electronics and Telecommunications*, vol. 65, no. 1, pp. 103–109, 2019.
- [13] Y. Hamad, A. Emad, and A. D. Arafat, "Hybrid cognitive-radio NOMA with blind transmission mode identification and BER Constraints," *TechRxiv. Preprint*, 2021.
- [14] T. N. Tran, D. T. Do, and M. Voznak, "On outage probability and throughput performance of cognitive radio inspired NOMA relay system," *Information And Communication Technologies And Services*, vol. 16, no. 4, pp. 501–512, Dec. 2018.
- [15] X. Wang, Z. Na, K. Y. Lam, *et al.*, "Energy efficiency optimization for NOMA-Based cognitive radio with energy harvesting," *IEEE Access*, vol. 7, pp. 139172 – 139180, Sep. 2019.
- [16] X. He, Y. Song, Y. Xue, *et al.*, "Resource allocation for throughput maximization in cognitive radio network with NOMA," *Computers, Materials & Continual*, vol. 70, no. 1, 2022.
- [17] H. Wang, R. Shi, K. Tang, J. Dong, and S. Liao, "Performance analysis and optimization of a cooperative transmission protocol in NOMA-Assisted cognitive radio networks with discrete energy harvesting," *Entropy*, vol. 23, no. 785, 2021.
- [18] K. Tang and S. Liao, "Outage analysis of relay-assisted NOMA in cooperative cognitive radio networks with SWIPT," *Information*, vol. 11, no. 11, 2020.
- [19] Y. Yu, Z. Yang, Y. Wu, *et al.*, "Outage performance of NOMA in cooperative cognitive radio networks with SWIPT," *IEEE Access*, vol. 7, pp. 117308–117317, 2019.
- [20] T. A. Hoang, C. B. Le, and D. T. Do, "Security performance analysis for pair domain NOMA employing in cognitive radio networks," *Bulletin of Electrical Engineering and Informatics*, vol. 9, no. 3, pp. 1046–1054, 2020.
- [21] V. Aswathi and A. V. Babu, "Performance analysis of NOMA-based underlay cognitive radio networks with partial relay selection," *IEEE Transactions. on Vehicular Technology*, vol. 70, no. 5, pp. 4615–4630, 2021.
- [22] L. Lv, L. Yang, H. Jiang, T. H. Luan, and J. Chen, "When NOMA meets multiuser cognitive radio: Opportunistic cooperation and user scheduling," *IEEE Trans. on Vehicular Technology*, vol. 67, no. 7, pp. 6679–6684, July 2018.
- [23] Z. Yang, J. A. Hussein, P. Xu, G. Chen, Y. Wu, and Z. Ding, "Performance study of cognitive relay NOMA networks with dynamic power transmission," *IEEE Trans. on Vehicular Technology*, vol. 73, no. 21, pp. 2882–2887, Mar. 202.
- [24] Q. Li and S. Zhao, "Robust secure beamforming design for cooperative cognitive radio nonorthogonal multiple access networks," *Hindawi*, vol. 2021, Feb. 2021.
- [25] N. Nandan, S. Majhi, and H. C. Wu, "Secure beamforming for MIMO-NOMA based cognitive radio network," *IEEE Communications Letters*, vol. 22, no. 8, pp. 1708–1711, Aug. 2018.
- [26] W. Shin, M. Vaezi, B. Lee, D. J. Love, J. Lee, and H. V. Poor, "Coordinated beamforming for multi-cell MIMO-NOMA," *IEEE Communications Letters*, vol. 21, no. 1, pp. 84–87, Jan. 2017.
- [27] Y. Yu, H. Chen, Y. Li, Z. Ding, and L. Zhuo, "Antenna selection in MIMO cognitive radio-inspired NOMA systems," *IEEE Communications Letters*, vol. 21, no. 12, pp. 2658–2661, Dec. 2017.
- [28] H. S. M. Antony and T. Lakshmanan, "Secure beamforming in 5G-Based cognitive radio network," *Symmetry*, vol. 11, no. 10, 2019.
- [29] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Elsevier Inc., 2007, ch.7.
- [30] Hypergeometric2F1. [Online]. Available: <http://functions.wolfram.com>
- [31] H. T. Tran, "Hierarchical codebook using the last layer to improve the angular estimation in MIMO system," *ICT Express*, vol. 6, no. 4, 2020.

Copyright © 2023 by Hoai Trung Tran. This is an open access article distributed under the Creative Commons Attribution License (CC BY-NC-ND 4.0), which permits use, distribution and reproduction in any medium, provided that the article is properly cited, the use is non-commercial and no modifications or adaptations are made.



Hoai Trung Tran received the B.E degree from University of Transport and Communications (UTC) in 1997 and the Master degree from Hanoi University of Science and Technology (HUST) in 2000. He received the Ph.D. degree in telecommunication engineering from University of Technology, Sydney (UTS), Australia in 2008. He is currently lecturer at the UTC, Vietnam. His research interests are digital signal processing (DSP), applied information theory, radio propagation, MIMO antenna techniques and advanced wireless transceiver design. He can be contacted at email: trungth@utc.edu.vn.